

Exploration of Aspherical Ejecta Properties in Type Ia Supernova: Progenitor Dependence and Applications to Progenitor Classification

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ABSTRACT

Several explosions of thermonuclear supernovae (SNe Ia) have been found to exhibit deviations from spherical symmetry upon closer inspection. Examples are the gamma-ray lines from SN 2014J as measured by INTEGRAL/SPI, and morphology information from radioactive isotopes in older remnants such as Tycho. A systematic study on the effects of parameters such as ignition geometry and burning morphology in SNe Ia is still missing. We use a 2D hydrodynamics code with post-processing nucleosynthesis and simulate the double detonations in a sub-Chandrasekhar mass carbon-oxygen white dwarf starting from the nuclear runaway in the accumulated He envelope towards disruption of the white dwarf. We explore potential variety through four triggering scenarios that sample main asymmetry drivers. We further investigate their global effects on the aspherical structure of the ejecta based on individual elements. We apply the results to the well observed SN 2014J and other recently observed SN remnants in order to illustrate how these new observational data together with other observed quantities help to constrain the explosion and the progenitors of SNe Ia.

Keywords: (Stars:) supernovae: individual: SN2014J – Gamma rays: stars – Hydrodynamics – Nuclear reactions, nucleosynthesis, abundances – Supernova remnants

1. INTRODUCTION

1.1. *Observational Constraints on Type Ia Supernova Progenitors*

Type Ia supernovae (SNe Ia) have demonstrated a wide diversity (e.g., Taubenberger 2017). To trace the progenitor and the explosion mechanisms, supernova light curves and spectra are indispensable. (See Appendix for the summary of the progenitors and the explosion mechanism.) Explosion features can be extracted from the light curve shape (e.g., Blondin et al. 2018), spectral lines such as Ca II and Ni II (e.g., Wilk et al. 2018) and the polarization (e.g., Bulla et al.

2016). The element abundance ratio can be the discriminant of current explosion models (e.g., Seitenzahl et al. 2009), by indicating the isotopic abundance ratios (e.g. $^{57}\text{Ni}/^{56}\text{Ni}$, $^{55}\text{Fe}/^{56}\text{Ni}$, $^{44}\text{Ti}/^{56}\text{Fe}$) and element abundance ratios (e.g., Mn/Fe and Ni/Fe; see Mori et al. 2018). These techniques had been applied to a few well-observed SNe Ia including SNe 2011fe, 2012cg, 2014J, 2015F and supernova remnants (SNR) 3C 397 (see recent works in for example Yamaguchi et al. (2015); Leung & Nomoto (2017a); Dave et al. (2017); Shen et al. (2018); Leung & Nomoto (2018); Zhou et al. (2020)). The progenitor mass and metallicity are constrained by this method, which allows further identification of isotopic mass fractions of radioactive Ni isotopes in SN 2012cg (Graur et al. 2016) and SN 2014J (Yang et al. 2018), and stable Ni by the nebular IR spec-

tra (see [Dhawan et al. 2018](#), for the application to SN 2014J).

Radioactive isotopes with a longer half life, e.g., ^{44}Ti with a half year of 59 years, can help to distinguish the different subclasses of SNe and their explosion mechanisms, by their effects on the decline rate of the late time light curve and the measuring the absolute abundances (see e.g., [Fry et al. 2015](#))).

Based on the tomography and morphology of SNR, the progenitor constraints are cast on SNR Tycho ([Wang & Li 2014](#); [Lopez et al. 2015](#)). The S and Fe spectral lines obtained from SNR 0519-69.0, 0509-67.5, and N103B are further examined in the shock-heated nebulae ([Seitenzahl et al. 2019](#)). The heated matter can emit X-ray lines which provide direct constraints on the explosion mechanism, in particular the asphericity of SNe Ia.

Analysis of SNRs on larger scales including the Small and Large Magellanic Cloud can reveal the SN explosion history ([Maggi et al. 2016, 2019](#)). Study of particular elements e.g., Mn, can point out the major explosion mechanism ([McWilliam et al. 2018](#); [de los Reyes et al. 2020](#); [Kobayashi et al. 2020](#)).

The morphology reveals the inherent explosion asymmetry. For individual SNRs, the possible explosion progenitors can be inferred from their X-ray emission, which reveals the chemical abundances ([Badenes et al. 2005, 2006, 2008](#)). Differences in the interaction between the SN ejecta and the ambient medium can also point out the progenitor ([Martínez-Rodríguez et al. 2018](#)). History of ejecta interaction with circumstellar matter (CSM) is contained in the late-time light curve as well ¹.

1.2. Our Previous Studies of SNe Ia and Present Work on Asymmetries

We have performed nucleosynthesis surveys of SN Ia explosions by the near-Chandrasekhar mass white dwarf model using the turbulent deflagration model with the deflagration-detonation transition in [Leung & Nomoto \(2018\)](#) (Paper I), the sub-Chandrasekhar mass white dwarf model using the double detonation model in [Leung & Nomoto \(2020a\)](#) (Paper II), and the near-Chandrasekhar mass model for SNe Iax using the pure turbulent deflagration model [Leung & Nomoto \(2020b\)](#) (Paper III). In these papers, we have conducted an extended parameter survey of the SN Ia models aiming at understanding the implication of its observed diversity

to progenitor model diversity, as well as constraints on explosion physics.

In Paper I, we considered models with the mass M of $1.30 - 1.38 M_{\odot}$ and metallicity of $Z = 0 - 5 Z_{\odot}$. The super-solar metallicity is suggested to explain the observed SNe Ia, including the SN remnant 3C 397 ($Z \sim 5Z_{\odot}$) and SN 2012cg ($Z \sim 3 - 5Z_{\odot}$). In Paper II, we consider WD models with $M = 0.9 - 1.3 M_{\odot}$, $Z = 0 - 5 Z_{\odot}$, and the He envelope mass of $M_{\text{He}} = 0.05 - 0.2 M_{\odot}$. We show that sub-Chandrasekhar mass models can also explain the isotopic ratio of nearby SNe Ia as the near-Chandrasekhar mass model does, but its Mn production cannot explain the nearby [Mn/Fe] trend taken from stars in the solar neighbourhood. Explosions of WD models with $M \sim 1.2 M_{\odot}$ can provide the key to distinguish the two explosion channels. In Paper III, we specifically model SNe Iax with the pure turbulent deflagration modes of the WDs for masses of $1.30 - 1.39 M_{\odot}$ and $Z = Z_{\odot}$ [Leung & Nomoto \(2020b\)](#).

In the present paper IV, we study the asymmetry of the WD models as suggested from SN 2014J, SNR Tycho, and other features mentioned above. The ejecta geometry is primarily dependent on the explosion progenitor. The near-Chandrasekhar mass model exploding by the turbulent deflagration model with the deflagration-detonation transition tends to explode spherically (see, e.g., [Roepke et al. 2007](#), for recent three-dimensional realizations). On the contrary, the sub-Chandrasekhar mass model which explodes by the He-induced double detonations can generate large-scale asymmetry because of the off-center trigger of the explosion (see, e.g., [Moll & Woosley 2013](#); [García-Senz et al. 2018](#); [Gronow et al. 2020](#), for some recent three-dimensional realizations showing aspherical structures).

We thus focus on the sub-Chandrasekhar mass models, which tend to have a more aspherical structure than the near-Chandrasekhar mass models. We note that in the literature there is no extensive work examining how sub-Chandrasekhar mass models exhibit a large-scale asphericity. In this Paper IV, we study for the first time how the different detonation mechanisms of the sub-Chandrasekhar mass model can generate the ejecta deviated from the canonical spherical model by multi-dimensional simulations.

Specifically, we will clarify how the detonation triggered in the He-envelope affects the ^{56}Ni distribution and ejecta structure in both position and velocity spaces. We also try to understand the underlying principles for the observed irregularities in SNRs.

We choose two-dimensional models so that we may test a larger number of models than three-dimensional models to uncover the global trend of the parameter

¹ We refer the interested readers to the related presentations in the conference "Progenitors of Type Ia Supernovae" available in http://bps.ynao.cas.cn/xzzx/201908/t20190820_510006.html

landscape. Also, quasi-spherical SN 2014J (see §4) has encouraged us to explore models with a certain level of symmetry (e.g., rotation symmetry and reflection symmetry) assumed in two-dimensional model, instead of arbitrary models without explicit symmetry as in three-dimensional models.

Ideally, three-dimensional models are naturally desired to match all features self-consistently. However, they are much more computationally expensive. As an exploratory study, we aim at searching the key properties in the detonation setting for the models to contain different features that might ultimately imprint in observational data. We want to understand what kind of shock interaction is necessary for generating the observed features, from which we may obtain hints on the initial detonation pattern. This will guide future three-dimensional simulations in setting up accurate initial models aiming for explaining SN 2014J or other supernovae.

1.3. Paper Structure

In Section 2, we describe our methodology and the models to be presented in this article. Then, we briefly review how we compute the explosion models of the sub-Chandrasekhar mass WD as the progenitor and we present the stellar parameters, explosion energetics, and the essential nucleosynthetic products.

In Section 3, we first present how the near-Chandrasekhar and sub-Chandrasekhar mass WDs differ by their large-scale asymmetry. Then we examine in details how the explosion ejecta and its chemical composition depend on the viewing (ejecta) angle. We also predict the expected morphology by extracting the representative elements.

In Section 4, we present a detailed case study for SN 2014J to show how the asymmetry and other nucleosynthetic yields can be used for constraining the explosion mechanism and the progenitors of SNe Ia. We cast constraints on the progenitor mass, initial explosion geometry, He envelope mass, and its metallicity.

In Section 5, we further apply our results on some recently observed SN remnants reported in the literature to demonstrate how the geometry can provide us the hints.

In Section 6, we discuss how this work is related to other hydrodynamics simulations in the literature.

2. METHODS AND MODELS

We use our two-dimensional hydrodynamics code developed for modeling the explosion models in this work. The code is based on high-order shock capturing scheme and time-discretization scheme, coupled with sub-grid

scale turbulence models, flame tracking schemes and nuclear reaction networks of arbitrary sizes. We refer the interested reader to the instrumentation paper which describes the prototype (Leung et al. 2015a). We have further extended the code to accommodate the code in different explosion scenarios. Different extensions are described in details for

- (1) SNe Ia in Leung & Nomoto (2017a, 2018, 2020a),
- (2) electron capture SNe in Leung et al. (2020); Leung & Nomoto (2017b, 2019); Zha et al. (2019b), and
- (3) dark matter admixed compact objects in Leung et al. (2015b, 2019); Zha et al. (2019a).

2.1. Input Physics

We follow Leung & Nomoto (2020a) for SN Ia using the sub-Chandrasekhar mass WD models. We solve the two-dimensional Euler equations in cylindrical coordinates. We further assume reflection symmetry of the x - y plane so that we model only one quadrant of the star. We use a realistic Helmholtz EOS (Timmes 1999) for describing the matter with free electrons with arbitrary relativistic level and degeneracy, nuclei as a classical ideal gas, photons with Planck distribution and electron-positron pair effects. Different from the near-Chandrasekhar mass WD models, we include:

- (1) the notation of He-detonation in the simulation,
- (2) its energy generation prescription and timescale, and
- (3) its propagation velocity.

We use the same solver for matter in the nuclear statistical equilibrium (NSE) as it is independent of the original composition of the matter, and only depends on the final density, temperature and electron fraction. Level-set methods are used for tracing the contour of the He- and C-detonation fronts. In Paper II, we have further performed a set of tests to justify the functionality of the code in the appendix. They include that

- (1) the C-detonation trigger is independent of the symmetry boundary we used²,
- (2) the explosion energetics are insensitive to the resolution used,
- (3) the shock convergence is less sensitive to the resolution, and

² We explored whether modeling the two-bubble structure using a hemisphere and a quadrant gives rise to different results. We showed that indeed the detonation waves collide identically as if they are laminar wave at the boundary where reflective boundary is imposed. In fact, this scenario provides a more stringent test to how robust the second detonation can be triggered because there is no geometric convergence taking place near the reflective boundary. As a result, the required He envelope mass predicted by this assumption is the upper limit. This value can be drastically reduced as geometric effects become stronger.

(4) the threshold of the detonation trigger is independent on the resolution.

We have further shown that in Paper II that our two-dimensional models give agreeing results with contemporary one-, two- and three-dimensional models found in the literature.

2.2. Post-Process Nucleosynthesis

In order to keep track of the detailed nucleosynthesis for constructing the isotope distribution, we use the tracer particle scheme (Travaglio et al. 2004; Seitenzahl et al. 2009; Townsley et al. 2016). It makes use of the massless Lagrangian tracers. They follow the fluid motion and record their own thermodynamical trajectories. The particles are "massless" that they do not change the fluid motion. After the hydrodynamical simulations, the tracers are post-processed with a large nuclear reaction network (a 495-isotope network containing isotopes from ^1H to ^{91}Tc Timmes et al. 2000). After post-processing, the spatial distribution of specific isotopes such as ^{16}O , ^{28}Si and ^{56}Ni are extracted for this data analysis.

2.3. Models

In this work, based on the formalism of our previous works, we examine further SN Ia models in the range between $0.95 - 1.0 M_{\odot}$ and we extend systematically to different initial He-detonation structures. We evolve WDs from the onset of He-detonation until no significant exothermic reactions take place. This can be because the WD is disrupted completely by both types of detonation, or the He-detonation fails to trigger the second detonation and quenches. In Table 1 we list initial densities and temperature used in our models. As indicated by observations data, we consider the He-envelope mass M_{He} from $0.05 - 0.10 M_{\odot}$.

In the table, we also list other related parameters including the central density ρ_c and the interface density ρ_{He} . Among all our considered models, the densities range from $\sim 10^7$ to 10^8 g cm^{-3} while the He-interface is from $\sim 10^5$ to 10^6 g cm^{-3} . Notice that this allows a major part of the star to carry out complete burning from CO matter to ashes in NSE at a density $> 5 \times 10^7 \text{ g cm}^{-3}$.

In general, three types of events can result. "N/A" means that no C-detonation occurs: this happens when the He-detonation is too weak (without the possible geometric convergence) to heat the CO matter to the sufficient temperature, or collision to create the required penetration. Results "cen" and "off" stand for centered and off-centered detonation, respectively. We also list the yielded ^{56}Ni and ^{57}Ni masses, obtained at the end

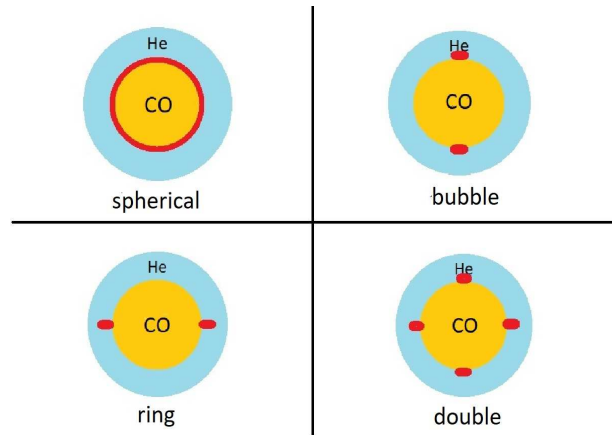


Figure 1. Graphical illustrations of the initial He-detonation configuration used in this article. The figure depicts the cross-sections of initial CO WDs with He-envelopes and the initial He-detonation put in by hand. The "S"- (spherical), "B"- (bubble), "R"- (ring) and "D"- (double) Types of geometry are presented. The orange and light blue regions stand for the CO-rich and He-rich regions. The red region stands for the zone which is assumed to be burnt already at the beginning of the simulations.

of simulations, where most exothermic reactions have ceased.

In Figure 1 we depict the four scenarios used in this work. They include a bubble ("B"-Type), a ring ("R"-Type), a bubble and a ring ("D"-Type) and a sphere ("S"-Type). This spans the possible initial He-runaway from the lowest symmetry to the highest symmetry. In the figure the cross-sections of the WD progenitor are drawn. The "B"-Type detonation corresponds to two bubbles, one at the "north"-pole. and one at the "south"-pole.

In order to realize the one-bubble event, simulations modeling the hemisphere explicitly is necessary. We remind that the C-detonation is triggered is very similar to the "R"-Type since, in this configuration, the C-detonation always starts after the shock converges at the opposite "pole" from where the detonation is initialized. Therefore, we may refer to "R"-Type series to trace how the detonation takes place. We note that, computationally, the detonation trigger is identical as indicated by Appendix B in Paper II.

3. REPRESENTATION OF ASPHERICITY OF SNE IA

In this section we examine how the asphericity of a SN Ia can be embodied by their observables. We first compare the typical ejecta structure from our two-dimensional simulations for the near-Chandrasekhar and sub-Chandrasekhar models to see how the progenitor mass affects the asphericity. Then we focus on the

Table 1. Initial conditions, explosion energetics and the global nucleosynthetic results of the sub-Chandrasekhar mass SN Ia models presented in this work. Models in bold font text are those with a comparable ^{56}Ni mass with SN 2014J. M , M_{He} and M_{CO} are the masses of the initial WD, He envelope, and CO core in units of solar mass. The column "flame" stands for the initial He-runaway geometry. R and R_{He} are the radii of the initial WD and the interface from the CO core to the He envelope in units of km. ρ_c and ρ_{He} are the densities in the core and in the CO-He interface. The columns "runaway?" and "type" represent whether the model develops the C-detonation and how the C-detonation is triggered (see also in the main text for the definition). "cen" and "off" stand for centered and off-centered C-detonation, while N/A means that no second detonation occurred. $M(^{56}\text{Ni})$ and $M(^{57}\text{Ni})$ are the final ^{56}Ni and ^{57}Ni computed by post-processing nucleosynthesis in units of M_{\odot} .

Model	M	M_{He}	M_{CO}	flame	R	R_{He}	ρ_c	ρ_{He}	2 nd runaway?	type?	$M(^{56}\text{Ni})$	$M(^{57}\text{Ni})$
095-050-B	0.95	0.05	0.90	B	6710	4760	2.23	0.06	No	N/A	N/A	N/A
095-050-R	0.95	0.05	0.90	R	6710	4760	2.23	0.06	Yes	off	0.11	3.04×10^{-3}
095-050-D	0.95	0.05	0.90	D	6710	4760	2.23	0.06	No	N/A	N/A	N/A
095-050-S	0.95	0.05	0.90	S	6710	4760	2.23	0.06	Yes	cen	0.45	1.14×10^{-2}
095-100-B	0.95	0.10	0.85	B	6710	4330	2.23	0.12	No	N/A	N/A	N/A
095-100-R	0.95	0.10	0.85	R	6710	4330	2.23	0.12	Yes	off	0.31	8.65×10^{-3}
095-100-D	0.95	0.10	0.85	D	6710	4330	2.23	0.12	Yes	off	0.29	8.63×10^{-3}
095-100-S	0.95	0.10	0.85	S	6710	4330	2.23	0.12	Yes	cen	0.48	1.28×10^{-2}
100-050-B	1.00	0.05	0.95	B	6180	4350	3.21	0.09	No	N/A	N/A	N/A
100-050-R	1.00	0.05	0.95	R	6180	4350	3.21	0.09	Yes	off	0.31	8.16×10^{-3}
100-050-D	1.00	0.05	0.95	D	6180	4350	3.21	0.09	Yes	off	0.08	2.31×10^{-3}
100-050-S	1.00	0.05	0.95	S	6180	4350	3.21	0.09	Yes	cen	0.60	1.60×10^{-2}
100-100-B	1.00	0.10	0.90	B	6180	3980	3.21	0.16	Yes	off	0.35	1.14×10^{-2}
100-100-R	1.00	0.10	0.90	R	6180	3980	3.21	0.16	Yes	off	0.46	1.30×10^{-2}
100-100-D	1.00	0.10	0.90	D	6180	3980	3.21	0.16	Yes	off	0.44	1.26×10^{-2}
100-100-S	1.00	0.10	0.90	S	6180	3980	3.21	0.16	Yes	cen	0.62	1.74×10^{-2}
105-050-B	1.05	0.05	1.00	B	5300	4110	4.33	0.10	No	N/A	N/A	N/A
105-050-R	1.05	0.05	1.00	R	5300	4110	4.33	0.10	Yes	off	0.48	1.24×10^{-2}
105-050-D	1.05	0.05	1.00	D	5300	4110	4.33	0.10	Yes	off	0.48	1.37×10^{-2}
105-050-S	1.05	0.05	1.00	S	5300	4110	4.33	0.10	Yes	cen	0.76	1.63×10^{-2}
105-100-B	1.05	0.10	0.95	B	5300	3730	4.33	0.19	Yes	off	0.49	1.65×10^{-2}
105-100-R	1.05	0.10	0.95	R	5300	3730	4.33	0.19	Yes	off	0.59	1.78×10^{-2}
105-100-D	1.05	0.10	0.95	D	5300	3730	4.33	0.19	Yes	off	0.59	1.80×10^{-2}
105-100-S	1.05	0.10	0.95	S	5300	3730	4.33	0.19	Yes	cen	0.70	2×10^{-2}
110-050-B	1.10	0.05	1.05	B	4930	3800	6.17	0.13	No	N/A	N/A	N/A
110-050-R	1.10	0.05	1.05	R	4930	3800	6.17	0.13	Yes	off	0.68	1.90×10^{-2}
110-050-D	1.10	0.05	1.05	D	4930	3800	6.17	0.13	Yes	off	0.60	1.72×10^{-2}
110-050-S	1.10	0.05	1.05	S	4930	3460	6.17	0.13	Yes	cen	0.82	1.90×10^{-2}
110-100-B	1.10	0.10	1.00	B	4930	3460	6.17	0.24	Yes	off	0.61	2.10×10^{-2}
110-100-R	1.10	0.10	1.00	R	4930	3460	6.17	0.24	Yes	off	0.75	2.37×10^{-2}
110-100-D	1.10	0.10	1.00	D	4930	3460	6.17	0.24	Yes	off	0.70	2.21×10^{-2}
110-100-S	1.10	0.10	1.00	S	4930	3460	6.17	0.24	Yes	cen	0.81	2.43×10^{-2}
Observations												
lower limit	0.7	0.03									0.4	2.32×10^{-2}
upper limit	3.1	0.09									0.7	5.53×10^{-2}

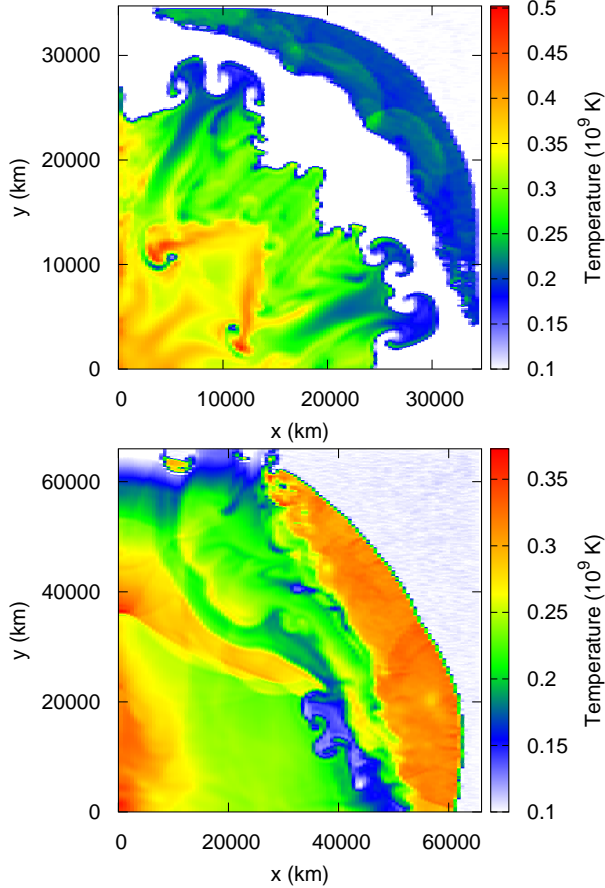


Figure 2. (top panel) The colour temperature profile of our benchmark SN Ia models from the explosion of a near-Chandrasekhar mass WD at ~ 4 s after the nuclear runaway has started (see Paper II for more details). (bottom panel) Similar to the top panel, but from a sub-Chandrasekhar Mass WD (this work).

sub-Chandrasekhar mass model to show how the initial detonation affects the ejecta distribution, velocity and remnant morphology.

3.1. Near-Chandrasekhar Mass WD versus sub-Chandrasekhar Mass WD

In this work we focus on the asymmetry of sub-Chandrasekhar mass WD. We do not focus on the near-Chandrasekhar mass WD because we observe that typical SN Ia models from the near-Chandrasekhar mass WD tends to have a more spherical structure. Here we present a comparison to outline the large-scale asymmetry in both models. For details of these models we refer interested readers to Papers I, II, and III for more detailed description and implementation.

The two models are chosen from our benchmark models. Both models are chosen to represent the “normal” SNe Ia by its ^{56}Ni production $\sim 0.6 M_{\odot}$. Furthermore the near-Chandrasekhar mass model also produces the

necessary amount of ^{55}Mn sufficient for reproducing the trends of stellar abundance near the solar neighbourhood.

In the top panel of Figure 2 we show the temperature in a section through the star of the near-Chandrasekhar mass model at ~ 4 s after the explosion. The model assumes turbulent deflagration with deflagration-detonation transition. The initial nuclear runaway assumes a centered deflagration wave with angular perturbations. The near-Chandrasekhar mass WD has a mass $1.37 M_{\odot}$ and a central density $3 \times 10^9 \text{ g cm}^{-3}$. A centered-flame *c3* (with three-finger structure) is used as the initial nuclear runaway. The finger structure is used to enhance hydrodynamical instabilities for the asphericity. The sub-Chandrasekhar mass model has a total mass $1.1 M_{\odot}$ with a helium envelope mass $0.1 M_{\odot}$ and a central density $\sim 6 \times 10^7 \text{ g cm}^{-3}$. A one-bubble configuration is placed along the rotation axis.

For our model, at $t \sim 4 - 5$ s the global distribution of ejecta structure begins to be frozen. Secondary features including Rayleigh-Taylor instabilities (“mushrooms”) still continue to grow. But most features are smoothed by the expansion, leaving a surface close to spherical. We can observe that by the external detonation transition, the detonation wave always wraps around the aspherical ash produced during deflagration. Surface asphericity in temperature appears to be significant in the Chandrasekhar mass model at a radius ~ 30000 km. However, the matter at the surface is mostly C and O. The area of interest, where iron-peak elements are synthesized, does not show much asphericity. They are produced primarily in the core, which means it is unlikely for large-scale asymmetry features from ^{56}Ni and ^{56}Co can be generated and be exposed. In this work, the core corresponds to the CO-rich matter under the He-envelope.

In the bottom panel of Figure 2, we show the temperature profile of a sub-Chandrasekhar mass CO white dwarf with a He envelope for comparison. The sub-Chandrasekhar mass model show a more explicit large-scale asphericity. The detonation is triggered near the “equator”. As a result, the He-detonation is stronger near the “equator”, as shown by the hot spot in 60000 km closer to the symmetry axis. Since the C-detonation propagates from the place near “equator” to the center and then outward. The inner ejecta from C-detonation has a preferred direction along the rotation axis. The high velocity flow along this direction will be responsible for the later large-scale asphericity.

3.2. Detonation Geometry Induced Asphericity

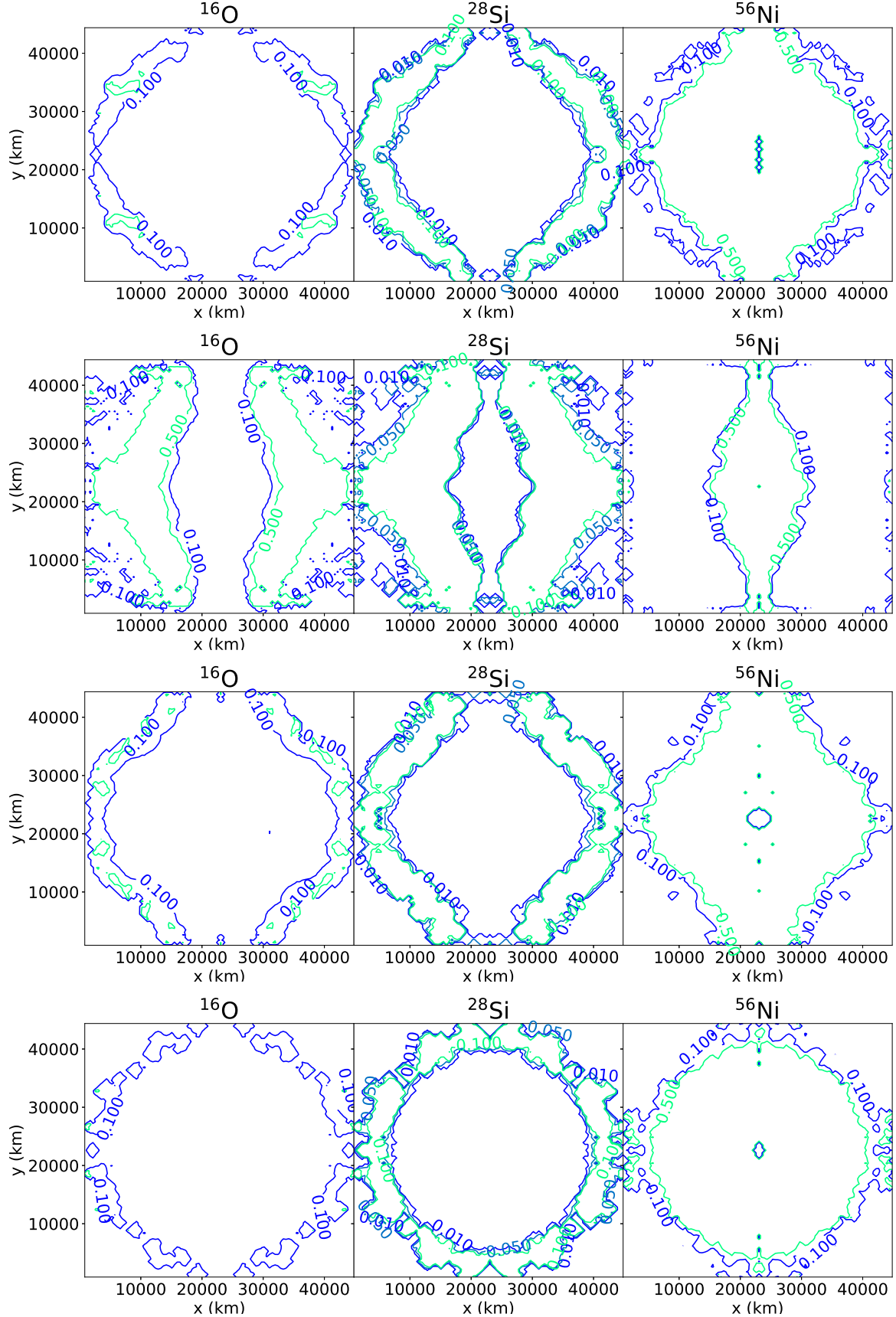


Figure 3. Mass fraction distributions of ^{16}O , ^{28}Si and ^{56}Ni for the explosion models 100-100-R (top panel), 100-100-B (second top panel), 100-100-D (third) and 100-100-S (bottom panel). The numbers stand for the corresponding contours of the mass fraction for the particular isotope.

Having shown that the sub-Chandrasekhar mass WD is capable of generating large-scale asymmetry, we further examine how the distribution of ^{56}Ni deviates from spherical symmetry. In Figure 3 we show the final distributions of the tracer particles which are the mass fraction contours of ^{16}O , ^{28}Si and ^{56}Ni . As examples, we consider models 100-100-R, 100-100-B and 100-100-S.

For Model 100-100-S, we can see that all the Ni-rich matter is covered by the thick envelope. We choose this series because all these models produce a comparable ^{56}Ni mass as a normal SN Ia.

The "S"-Type model does not show any significant amount of ^{56}Ni near the surface. The distribution confirms that the initial spherical symmetry makes the detonation wave propagate only spherically. Except for small scale perturbation coming from discretization effects from cylindrical coordinate to spherical coordinate, the large-scale distribution is to a good approximation spherical. The detonation waves do not collide with each other except at the stellar core when the detonation waves converge. There is no detonation wave interaction by shock collision or shock convergence in the He envelope. The amount of ^{56}Ni production becomes lower in the envelope. Hence, this model is less likely to explain the observed early ^{56}Ni decay line.

The "R"- and "B"-Type models show more near surface ^{56}Ni . The ^{56}Ni is ejected more along the polar direction for the "B"-Type detonation, meanwhile the "R"-Type detonation ejects matter more spherical, but in a more elongated manner compared with "S"-Type model. The "D"-Type model also behaves similarly to the "R"-Type except that the ^{56}Ni around the "equator" is lower in the abundance.

We remind that it is the asynchronous burning of helium, coupled with the geometric convergence creates the observed asphericity. The asynchronous burning allows He with the same initial density, to be burnt at different time. When the detonation time is delayed, the density of the He-matter decreases, thus making it less likely to generate ^{56}Ni in the first place. On the contrary, the geometric convergence creates the hot spot for triggering ^{56}Ni synthesis robustly. However, the effect of geometric convergence is more localized. In a similar way, the detonation wave collision can create the high temperature and high density zones for synthesizing ^{56}Ni .

3.3. Velocity Distribution of Ejecta

One of the features in SN2014J is the early time ^{56}Ni signal and time-dependent velocity for late-time ^{56}Co (see §4). It is therefore interesting to further study how the initial detonation configuration can give rise to the

diversity of the isotope velocity distributions. In Figure 4 we plot the ejecta distribution in the velocity space to analyze how the detonation affects the final ejecta distribution. The distribution is angular-averaged. More samples of SNe Ia show similar early pre-maximum bumps (see, e.g., Jiang et al. (2017, 2018)). The possibilities of these SNe Ia forming a sub-class showing that SN Ia with observable He-burning features can have a common evolutionary path. Furthermore, such early gamma-ray flux can be another important sign for future telescopes to capture the early optical evolution of these SNe (Wang et al. 2019).

For Model 100-100-B, the one bubble configuration allows ejecta to be concentrated with ^{56}Ni -rich material (see Figure 3) near the "equator". As a result, there is a multi-layered distribution of ^{56}Ni . Below 6000 km s^{-1} , the ejecta are filled with $^{56-58}\text{Ni}$. From 6000 to 8000 km s^{-1} , ^{28}Si and ^{32}S are the major isotopes. $8000 - 11000 \text{ km s}^{-1}$ unburnt oxygen is the major element and outside He is in the main element in the ejecta. We remark that the ^{56}Ni distribution is not monotonically decreasing, as compared to the classical spherical model. It first drops around 10000 km s^{-1} , showing that the detonation reaches the low-density region for C-burning. After that the mass fraction of ^{56}Ni rises again and quickly drops in its abundance. This shows that the shock collision in the He-envelope allows formation of ^{56}Ni directly. But the shock strength is not strong enough to channel the outburst of ^{56}Ni , as seen by the covering layer of ^4He .

For Model 100-100-D, the strong collision away from the axis allows an outburst of ^{56}Ni during ^4He -burning at early time. This is also reflected in the ejecta distribution. Ejecta with a velocity below 10000 km s^{-1} are again filled with Ni isotopes. ^{28}Si and ^{32}S are the major isotopes in the velocity range $10000 - 11000 \text{ km s}^{-1}$. Products of incomplete C-burning, such as unburnt ^{16}O , can be found most abundant up to 12000 km s^{-1} . Outside that ^{56}Ni and ^{16}O are the major isotopes. From this it can be seen that multiple ignitions allow ^4He to be burnt quickly along the α -chain. Such outermost ^{56}Ni can be readily ejected and be seen through its decay.

For Model 100-100-R, the geometric convergence takes place at the pole which is strong enough to create similar pinching to the He-envelope. The ejecta are covered with IMEs from $10000 - 11000 \text{ km s}^{-1}$ and ^{16}O from 11000 to 12000 km s^{-1} . The outermost layer is mixed with ^{56}Ni and ^4He with hints of ^{28}Si and ^{32}S . Similar to Model 100-100-D, a thin layer of IMEs between $10000 - 11000 \text{ km s}^{-1}$ and then incomplete C-burning products between $11000 - 12000 \text{ km s}^{-1}$. The outermost ejecta are a mixture of ^{56}Ni , ^4He , ^{28}Si , ^{32}S and ^{16}O .

For Model 100-100-S, the spherical detonation allows a stratified structure in the ejecta. Ejecta with a velocity below 12000 km s^{-1} are dominated by ^{56}Ni , ^{57}Ni and ^{58}Ni . IMEs including ^{28}Si and ^{32}S are mostly found between $12000 - 14000 \text{ km s}^{-1}$. Ejecta with a velocity $> 14000 \text{ km s}^{-1}$ are occupied by ^4He .

By comparing these four models, it demonstrates the possibility of mixing ^{56}Ni at high velocity, namely at the outermost ejecta for the "D" and "B"-Types. The oblique shock and the geometric convergence of detonation inside the He-envelope can provide the necessary thrust for channeling the ^{56}Ni produced in the He-detonation to the outermost part of ejecta. When the ejecta quickly expands, the gamma-rays produced by the decay of ^{56}Ni into ^{56}Co may be directly seen after the surface matter becomes optically thin.

3.4. Directional Dependence of Ejecta

In the previous section we have examined the angular averaged ejecta velocity composition and we show that the ejecta composition depends on the explosion geometry. Here we further on analyze the ejecta composition by choosing specific angular slices. In particular we choose the angular slices at $0 - 9 \text{ deg}$ from the rotation axis and from the symmetry axis (i.e. $81 - 90 \text{ deg}$ from the rotation axis) to contrast the ejecta composition. This demonstrates the difference between the metal production with or without geometric convergence.

3.4.1. Bubble-Type Explosion

The one-bubble configuration has shown to be weaker than the one-ring counterpart due to the lack of shock convergence. The explosion is weaker with a lower ^{56}Ni production. In the top left and right panels of Figure 5 we plot the ejecta distribution in the velocity space of Model 100-100-B for two angular slices, near the rotation axis (left) and near the symmetry axis (right).

The two slices show very different ejecta structures. The ejecta along the rotation axis is slightly faster than the rotation axis. It is because the initial detonation triggered along the "poles". However we remark that it does not mean there is more substances along the axis because the velocity space does not follow the mass coordinate directly. The iron-peak elements in the core also differ a lot. Along the "poles", the representatives of iron-peak elements including $^{56,57,58}\text{Ni}$ occupy the innermost $\sim 7000 \text{ km s}^{-1}$ and then the abundance quickly drops off. On the other hand, the iron-peak element-rich core extends up to 12000 km s^{-1} along the "equator". The large difference comes from the C-detonation. It is triggered near the "equator". As a result, the C-detonation first burns the material along the "equator" and then reaches the center, and then burns the matter

along the "equator". There is more time for the matter to move outwards and expands, thus yielding a weaker heating effect.

The IMEs form the middle layer from $7000 - 17000 \text{ km s}^{-1}$ along the "poles" and from $12000 - 15000 \text{ km s}^{-1}$ along the "equator". Along the "poles", as the velocity increases, which corresponds to lower density matter, some unburnt ^{12}C can be seen. However, there is no such trace along the "equator". Again, this demonstrates that the detonation along the "poles" is weaker than that along the "equator" due to the time lapse during expansion.

From $17000 - 21000 \text{ km s}^{-1}$ along the "poles" ^{56}Ni dominates the ejecta again. These are the product of the He-detonation as the ^4He mass fraction along becomes significant. They share similar mass fractions up to the surface. On the other hand, along the "equator" there is more ^{56}Ni from $15000 - 19000 \text{ km s}^{-1}$ and more ^4He from 19000 km s^{-1} onward.

3.4.2. Ring-Type Explosion

The one-ring configuration on the other hand shows stronger explosion than the one-ring counterpart through the shock convergence near the "poles". The explosion is also stronger with a higher ^{56}Ni production. In the bottom left and right panels of Figure 5 we plot the same as the top panels but for Model 100-100-R.

Along the "poles", the ^{56}Ni builds the core of iron-peak elements which extends up to 12000 km s^{-1} . Other iron-peak elements such as ^{57}Ni and ^{58}Ni are overwhelmed by the IMEs (^{28}Si and ^{32}S as two representatives) at a lower velocity of 10000 km s^{-1} . Beyond 12000 km s^{-1} , IMEs are the dominant species until the surface. A minor jump of ^{56}Ni can be seen only near the surface, unlike Model 100-100-B. ^4He is also much lower than the IMEs, and is $\sim 20\%$ of the surface abundance.

On the other hand, there is no such transition of ^{56}Ni -rich core to IME-envelope along the "equator". A rapid jump of IMEs appears near 9000 km s^{-1} . However, the total abundance is about $0.1 - 0.2$ lower than ^{56}Ni .

3.4.3. Model Comparison

The models 100-100-B and 100-100-R constitute two extremes of initial detonation configurations with the minimal perturbation from the spherical detonation. The distribution of elements in the velocity space appears to be very distinctive.

Without sufficient shock convergence, the explosion ejecta consists of much stronger traces of IMEs in the middle layer and ^{56}Ni - ^4He transition near the surface. The distribution of ^{56}Ni is discontinuous along some direction in the "B"-Type model, but continuous in

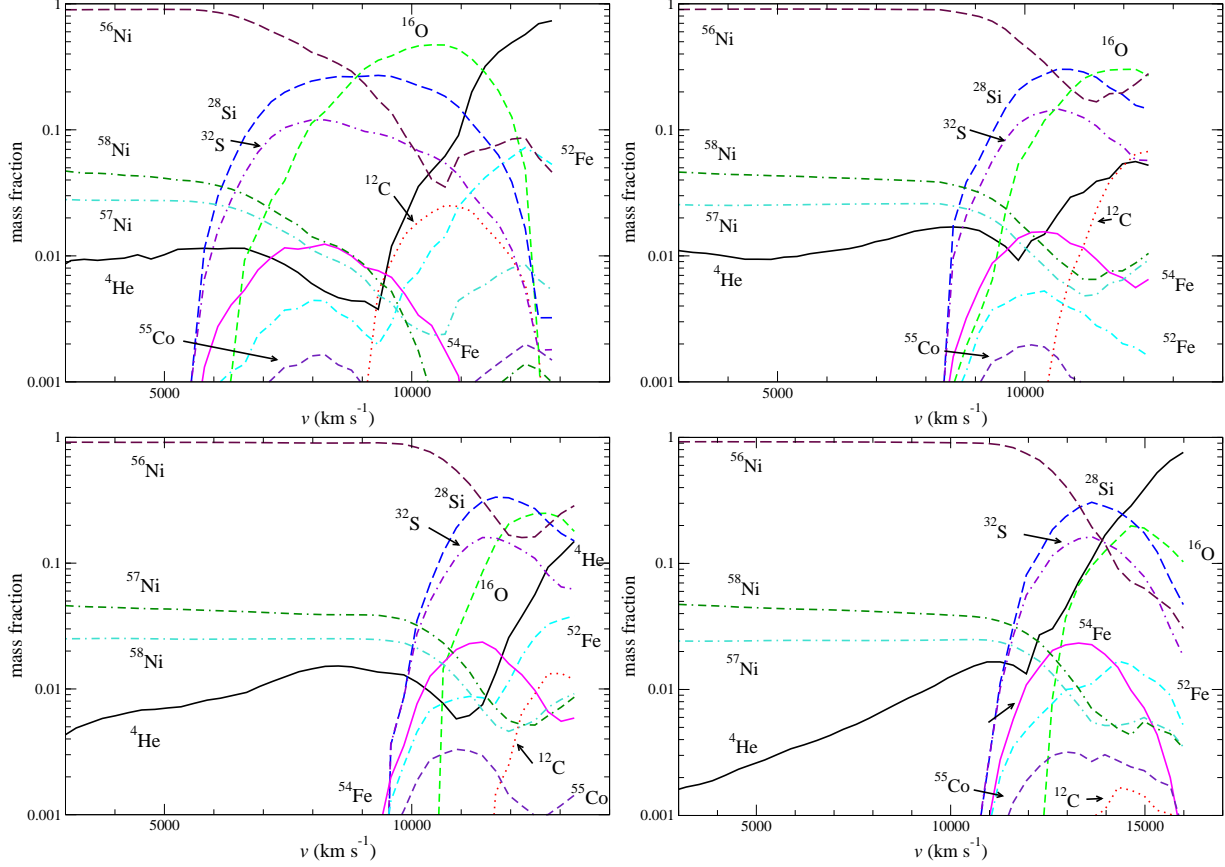


Figure 4. Velocity distributions of the ejecta for Models 100-100-B (top left), 100-100-D (top right), 100-100-R (lower left) and 100-100-S (lower right) respectively.

the "R"-Type model. Meanwhile, IMEs are more pronounced in the "R"-Type model near the surface but not the "B"-Type model. These distinctive features can be the indicator of where the initial He-detonation starts.

3.5. ^{56}Ni Mass: Dependencies on the WD Mass and Detonation Morphology

The explosion of a sub-Chandrasekhar mass WD is known to be sensitive to the initial WD mass, M . It is because the WD is degenerate, so that the central density varies largely from 10^7 g cm^{-3} (for $M \sim 0.95 M_\odot$) to 10^9 g cm^{-3} (for $M \sim 1.20 M_\odot$). The difference in the central density corresponds to a difference in the average density of WD matter, where the CO matter can be completely burnt to ^{56}Ni when it has a typical density $\sim 5 \times 10^7 \text{ g cm}^{-3}$. As a result, the amount of final ^{56}Ni drastically varies by a factor of 2 – 6 when M increases from 0.9 to $1.2 M_\odot$.

In Figure 6, we show the final ^{56}Ni mass against M for the four different detonation morphologies described in the previous subsections. Here the magenta box represents the range of ^{56}Ni derived from the gamma-ray data from SN 2014J. See §4 for the application to SN 2014J. Models in which the C-detonation cannot be trig-

gered are omitted, i.e. 095-050-B, 095-050-B, 100-050-B, 105-050-B, 110-050-B, and 095-050-D. All of these show a typical He nova event without the C-detonation (Kippenhahn & Thomas 1978; Piro & Bildsten 2004), which is inconsistent with the supernova observation that the whole star is disrupted after the explosion. From both figures, we can see that the "S"-series is the strongest, and then "D"- and "R"-series. The "B"-series is the weakest for the same initial M .

The possibility of triggering the C-detonation in the low M_{He} limit relies on the detonation symmetry. The CO WD models with a lower mass from $M = 0.95$ to $1.10 M_\odot$ show that, with the lowest symmetry ("B"-type), there is no geometric convergence. The only shock collision occurs when the detonation reaches the "equator" of the WD. The assumed boundary condition (reflection symmetry) allows the arriving shock waves to collide in a laminar way. As indicated in Paper II, when M increases, the minimum M_{He} required for triggering the second detonation decreases. According to the similar work without assuming reflection symmetry (e.g., Fink et al. 2014), the minimum necessary M_{He} drops from 0.126 to $0.0035 M_\odot$ when the CO WD mass in-

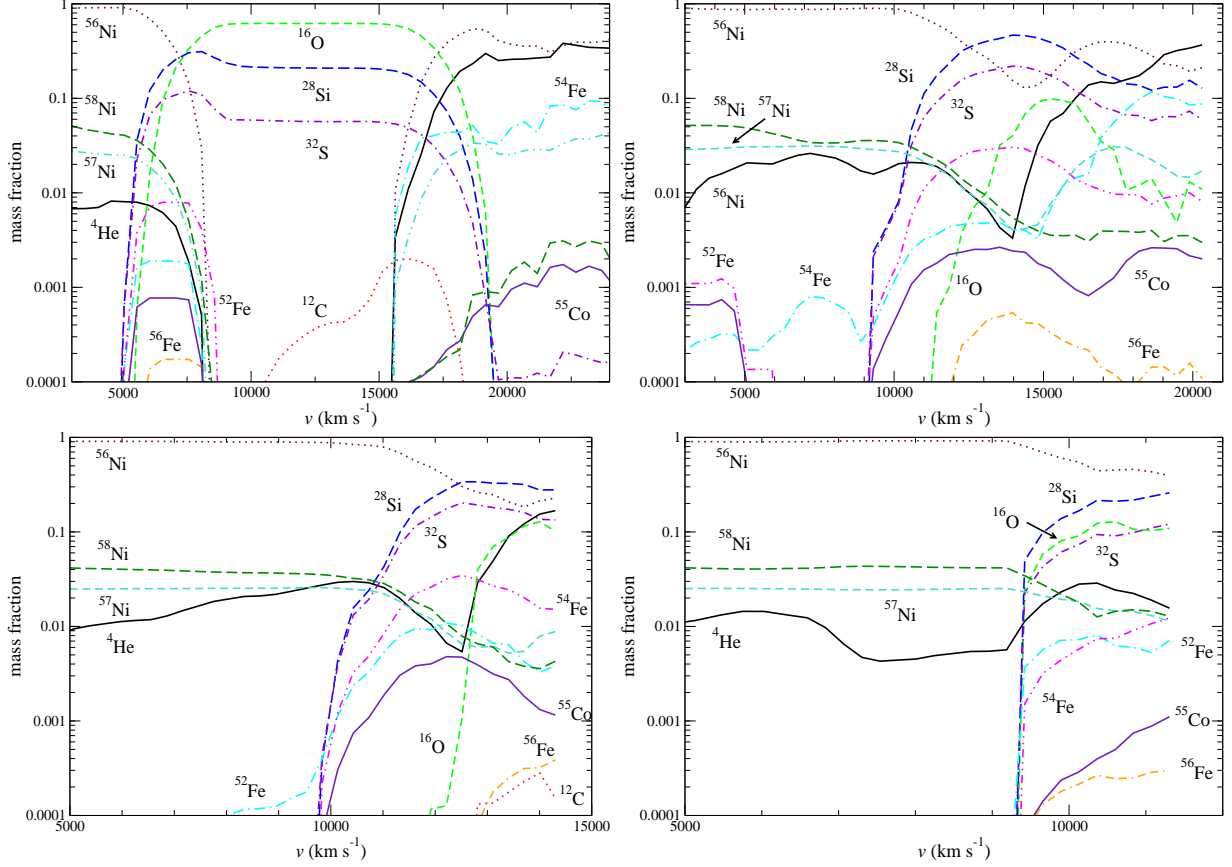


Figure 5. (top left panel) Ejecta compositions in the velocity space for Model 100-100-B along the angular slice from 0 – 9 deg from the rotation axis. (top right panel) Same as the top left panel, but for the angular slice 81 – 90 deg from the rotation axis. (bottom left panel) Ejecta compositions in the velocity space for Model 100-100-R along the angular slice from 0 – 9 deg from the rotation axis. (bottom right panel) Same as the bottom left panel, but for the angular slice 81 – 90 deg from the rotation axis.

creases from 0.810 up to 1.385 M_{\odot} . Both works give us an insight that a higher M_{He} is necessary to provide the sufficient shock strength in triggering the C-detonation.

By only considering the white dwarf mass, a higher M means that the transition from the CO core to the He envelopes takes place at a higher density. This increases the typical reaction rate and hence the energy production. The post-shock temperature in the He-envelope is, therefore, higher for more massive white dwarfs, where the burnt matter can reach the threshold temperature easier, independent of additional geometrical convergence. Thus, a higher M model favours the trigger of the second detonation.

We remind that, in simulations using a hemisphere of a WD, the dependence on the M_{He} is stronger than simulations using a quadrant. It is because the detonation starts from one pole and then the detonation wave wraps over the He-envelope and converges at the other pole. In this situation a shock convergence similar to the "R"-Type detonation always happens. As shown in the

table for the "R"-Type detonation, the corresponding minimum M_{He} for second detonation is lower.

For "D", "R" and "S", they have a higher symmetry where there is geometric convergence by means of oblique shock, two-dimensional shock convergence (from a ring to a point) and three-dimensional shock convergence (from a sphere to a point) respectively. The resultant temperature in the CO core can be much enhanced by the converged shock strength. Therefore, the minimum M_{He} required to trigger the second detonation is more relaxed.

The difference in the final ^{56}Ni mass for the same M at different He-detonation is related to the propagation of the C-detonation direction. The "S" model is always the strongest because the C-detonation begins at the center and propagates outward, so that most of the star remains approximately static before the detonation wave arrives. This ensures the matter remains less expanded and hence maintaining a higher density, which results in a stronger explosion. On the contrary, in the "B", "D" and "R" models, the off-center C-detonation means that

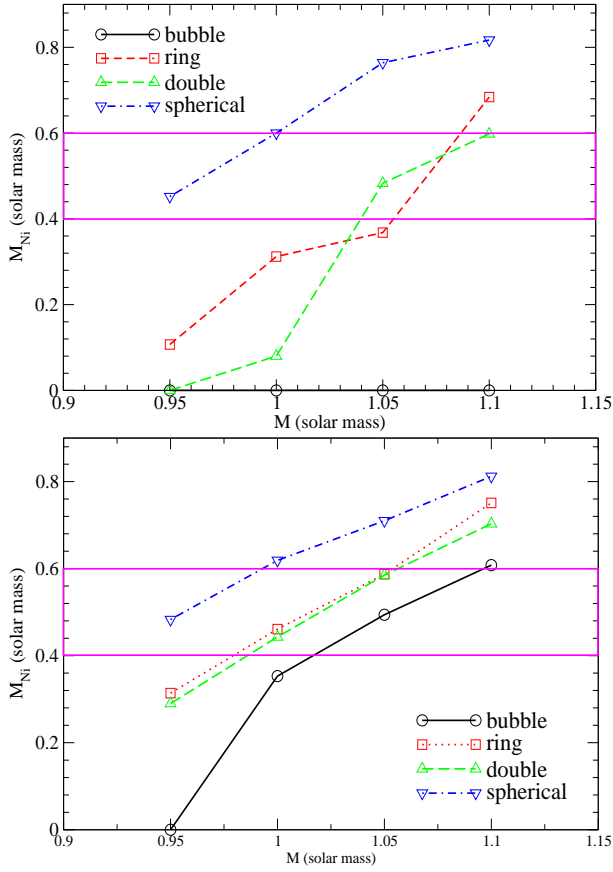


Figure 6. ^{56}Ni mass against M for the four different initial He-detonation configurations for $M_{\text{He}} = 0.05 M_{\odot}$ (upper panel) and $0.10 M_{\odot}$ (lower). The magenta box represents the range of ^{56}Ni derived from the gamma-ray data from SN 2014J (§4).

the C-detonation has to overcome the density gradient in order to reach the high density matter in the core. This implies that the relative explosion strength is weaker because of the density gradient.

3.6. Morphology of Remnants

The early ^{56}Ni in the ejecta near the surface provides distinctive hints on the asphericity of the detonation model. Furthermore, the distribution of iron-peak elements, as later they become heat-shocked by the backward shock in the ejecta, they can reach a temperature $\sim 10^6$ K, which is sufficient hot for X-ray emission for further diagnosis. This will give further constraints on the explosion models. For example, the X-ray spectra of the SN remnant 3C 397 has been used to a diagnosis of a Chandrasekhar mass WD progenitor (Yamaguchi et al. 2015; Leung & Nomoto 2018; Dave et al. 2017).

In Figure 7 we plot the element distribution of Cr, Mn, Fe and Ni for the Models 100-100-B, 100-100-R, 100-100-D and 100-100-S. We assume that after the star

reaches homologous expansion the distribution of ejecta remains mostly unchanged. However, we also remark that the radiation energy during decay of radioactive isotopes can still trigger inner motion and affect the element distribution. But the effect is secondary compared to the initial distribution during explosion.

The distribution of the iron-peak elements show more diversity than the major elements as shown in the previous section. From the contour shape we observe two features. First, Cr and Mn almost follow each other. And Fe and Ni follow each other. Second, Cr and Mn tends to have a more spherical distribution while Fe and Ni follow more closely to the detonation geometry.

Model 100-100-R shows the the largest deviation from a spherical structure among all four elements. Model 100-100-B has spherical Cr and Mn but aspherical Fe and Ni. Similar characteristics appear for Model 100-100-D. All elements are spherical in the Model 100-100-S, as anticipated by the initial spherical symmetry. To disentangle between Model 100-100-B and 100-100-D, we notice that the distribution of Cr is more irregular in Model 100-100-D, compared to the quasi-spherical distribution in Model 100-100-B. However, the difference is subtle.

4. CASE STUDY: APPLICATION TO SN 2014J

4.1. The Inspiring Case of SN2014J

SN 2014J is a special example of SNe Ia exploded in the nearby galaxy M82 just 3.3 Mpc away, the closest SN Ia in the last four decades. Its vicinity to from the Milky Way galaxy has provided the chance for detailed multi-band observations including the radio (Pérez-Torres et al. 2014), infrared, optical (Goobar et al. 2014; Kawabata et al. 2014), UV (Foley et al. 2014), X-ray (Terada et al. 2016) and gamma-ray (Siegert & Diehl 2015; Diehl et al. 2014, 2015; Diehl 2015; Churazov et al. 2015; Isern et al. 2016) bands, with its spectra at early and late time (Ashall et al. 2014; Jack et al. 2015; Dhawan et al. 2018).

Measurements of SN 2014J have been made in different works in the literature. In Churazov et al. (2014), the estimated $M_{^{56}\text{Ni}} = 0.56 \pm_{0.06}^{0.14} M_{\odot}$ and the estimated ejected mass is $1.2 \pm_{0.5}^{1.9} M_{\odot}$. A similar measurement is found in Diehl et al. (2015) which gives $M_{^{56}\text{Ni}} = 0.50 \pm 0.12 M_{\odot}$. For ^{57}Ni , Yang et al. (2018) reports that $^{57}\text{Ni}/^{56}\text{Ni}$ has a mass ratio $0.065 \pm_{0.004}^{0.005}$ based on the B-band maximum light and $0.066 \pm_{0.008}^{0.009}$ based on the pseudo bolometric light curve. Stable Ni mass is constrained at $0.053 \pm 0.018 M_{\odot}$ (Dhawan et al. 2018).

It shows gamma-ray features which agree well with the classical spherical pure deflagration model W7

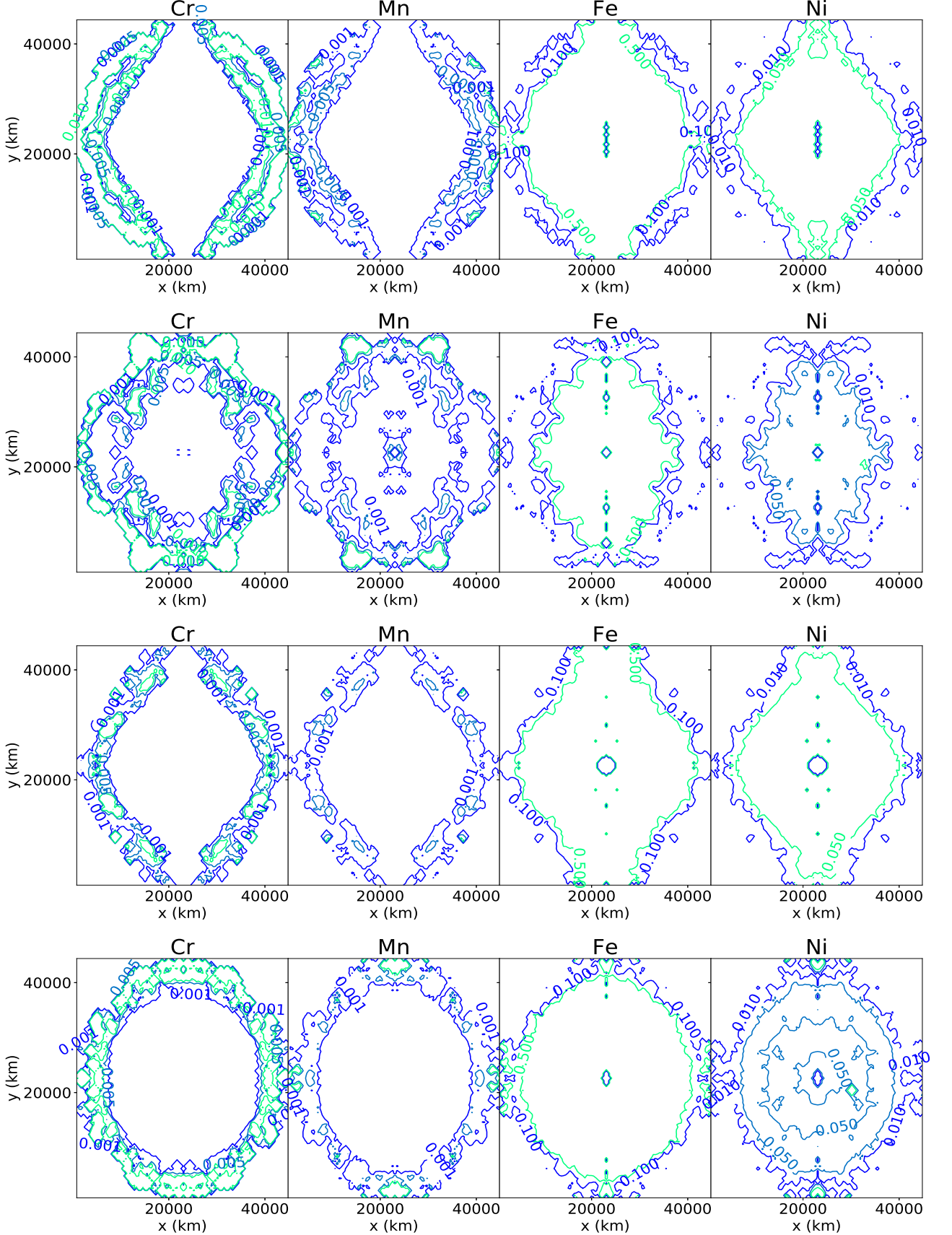


Figure 7. Stable element mass fraction distributions of the explosion models 100-100-B (1st row), 100-100-R (2nd row), 100-100-D (3rd row) and 100-100-S (4th row) for Cr, Mn, Fe and Ni for the panels from left to right. The numbers stand for the corresponding mass fraction contours for these isotopes.

(Churazov et al. 2014; Diehl et al. 2015). In photometry, SN 2014J appears to be a normal SN Ia (Isern et al. 2016) which shows a comparable structure with the W7 model (Nomoto et al. 1984), and a similar abundance profile with heavy elements (e.g., ^{56}Ni) in the core and lighter elements (e.g. Si and S) in the envelope (Ashall et al. 2014).

However, detailed examinations of the observational data of SN 2014J reveal some differences from ordinary SNe Ia. For example, the rise of UVOIR light curve with time (e.g. Nugent et al. 2011) shows its delay in SN 2014J. The early light curve of SN 2014J suggests a ‘shoulder’ only a few days after the first light (Goobar et al. 2014). The late time evolution (beyond few hundred days) shows derivations from classical SNe Ia such as SN 2011fe, where the slower decline rate suggests interactions with CSM (Foley et al. 2014; Yang et al. 2018). The ultraviolet data of SN 2014J shows large extinction (Brown et al. 2015). Such extinction and CSM can be in the dusk disk structure (Nagao et al. 2017). Images around SN 2014J do not show an observable companion star, thus making its companion as a red giant unlikely (Margutti et al. 2014; Pérez-Torres et al. 2014; Kelly et al. 2014).

It seems that the progenitor of SN 2014J is still a question of debate (e.g., Margutti et al. 2014; Pérez-Torres et al. 2014; Dragulin & Hoefflich 2016; Graur & Woods 2019). We note that these constraints on CSM cannot be applied to the presupernova environment of a uniformly rotating white dwarf with a slightly super-Chandrasekhar mass in the single degenerate scenario as calculated by Benvenuto et al. (2015). The gamma-ray signal cannot distinguish with high significance which class SN 2014J belongs to (Terada et al. 2016).

4.2. Aspherical Features of SN 2014J and Constraints on Models

According to further examinations, the observational data of SN 2014J show features which deviate from the spherical approximation as discussed below.

First, the early gamma-ray observations with INTEGRAL discovered the lines at 158 and 812 keV ($T_{1/2} = 6.6$ d) that are characteristic for the ^{56}Ni decay around 17.5 days after the inferred explosion date (Diehl et al. 2014; Isern et al. 2016). The model fits of Isern et al. (2016) appeared more consistent with a red-shifted and broadened ^{56}Ni emission, so that they suggested ejection of ^{56}Ni -rich material in a blob moving away from the observer. The analysis by Diehl et al. (2014) was performed in finer energy bins, and without any model bias; their sampling of possible spectral solutions sug-

gest a narrow emission line from ^{56}Ni at the laboratory energy value, with indications of blue- as well as red-shifted emission lines. This led them to suggest a model with ^{56}Ni ejected perpendicular to the observer’s line of sight. But the total significance of this surface ^{56}Ni line emission is only 3 – 4 sigma, hence both interpretations remain possible.

The second aspherical feature appears at later times. When the energy output is dominated by the decay of ^{56}Co ($T_{1/2} = 77.1$ d), the measured Doppler-shifts of the ^{56}Co decay lines show the expected behavior, plus an additional structure of (at least) three blobs of distinctive velocities (i.e. showing an early blue-shift when the red-shifted part is opaque in the line of sight, and then becoming symmetric with no Doppler-shift later as pointed out in Diehl et al. 2015). Instead, at least three distinctive centroid energies could be identified. This may correspond to fluid parcels containing ^{56}Co are ejected with different velocities with respect to the Earth frame. This time-dependent variations of the Co decay line frequency, denoted as flickering, is observed in SNe Ia for the first time. This suggests the possibility that ‘blobs’, or large scale asymmetries, developed during the explosion.

To summarize, SN 2014J has shown that the theoretical conflicts with the classical spherical model. These include: (1) the observed Doppler-broadened ^{56}Co lines, which have an irregular appearance with time (Diehl et al. 2015), (2) hints of the ^{56}Ni decay lines on the surface (Diehl et al. 2014), and (3) an enhanced ionization on the outer part of the star as revealed by the early atomic line spectra.

4.3. Near-Chandrasekhar Mass Models That Produce Surface ^{56}Ni

Among the above mentioned features of SN 2014J, the existence of ^{56}Ni near the surface suggested by the INTEGRAL data is the main motivation for the present study. The missing of the C and O absorption lines is possibly connected to the exposed ^{56}Ni lines (Goobar et al. 2014). It might be worth noting that these expected Ni lines are not seen in infrared at this time.

Before discussing the sub-Chandrasekhar mass models in the following subsections, we note that near-Chandrasekhar mass models have variations depending mainly on the mass accretion rate, and some models produce ^{56}Ni near the surface of WDs. In the classical picture of the near-Chandrasekhar mass model, ^{56}Ni is concentrated in the inner core (The & Burrows 2014). However, ^{56}Ni is produced near the surface in the following models.

Nomoto (1982a) showed several models where the WD mass increases to the near-Chandrasekhar mass with slow accretion of He. If the accretion rate is low enough, the accreted He is too cold to be ignited, thus being just accumulated near the surface. Eventually, the WD becomes massive enough to ignite the central C-deflagration.

In the *late* detonation model by Yamaoka et al. (1992), the deflagration-detonation transition can occur in the outer layer of the near-Chandrasekhar mass WD and burns He to produce ^{56}Ni near the surface (see model W7DHE in Fig. 3 of Yamaoka et al. 1992). Such a deflagration-detonation transition might occur at the very steep density gradient near the WD surface even with a small amount of He. The deflagration-detonation transition near the surface would be likely to occur in an aspherical manner which might be interesting for further study.

4.4. Modeling Issues on Sub-Chandrasekhar Mass Models

Although the production of ^{56}Ni near the surface is possible in the late detonation of the near-Chandrasekhar mass model (W7DHE), we here focus on the sub-Chandrasekhar mass models in order to apply the results of this Paper IV to the “asphericity” of SN 2014J.

SN 2014J was first proposed to be the ignition of a “He belt” accumulated in the orbital plane of the binary (Diehl et al. 2014). The He belt model tried to explain the early ^{56}Ni decay line detected with close-to-zero line Doppler shift. This picture can explain the origin of the early time ^{56}Ni decay line. However, on top of that, in Diehl et al. (2014) such decay line has a very small redshift. The He belt model, when observed from the “north-” or “south-poles”³, the ejection of Ni from the He envelope will be all along the “equator” direction. As a result, it provides the source of Ni with small Doppler shift. Also it remains unclear if the He belt can be formed and maintained stably during the binary accretion (Kippenhahn & Thomas 1978; Piro & Bildsten 2004).

In the following discussion, we search for a qualitative model that may resemble with the observed characteristics of SN 2014J. We note that the model asphericity should not be too strong, as otherwise it would violate the indicated proximity of SN 2014J with the classical

spherical model. Thus we set the He envelope in spherical form. To generate the required surface Ni, different He-detonation configurations are studied.

Our attempt is to elucidate the physical conditions which are able to reproduce the distinctive features of SN 2014J qualitatively. Owing to the complexity of multiple data constraints and the subtlety in the interpretations of different observations, we avoid to scrutinize an exact or complete model that can explain all quantitative features for SN 2014J. Our focus is, therefore, on the early ^{56}Ni line emission, the ^{56}Ni mass as derived from the peak (V-band) luminosity, large-scale asymmetries as implied by the flickering ^{56}Co decay line profiles, and the $^{57}\text{Ni}/^{56}\text{Ni}$ mass fraction ratio from the late-time light curve. Using $^{57}\text{Ni}/^{56}\text{Ni}$ to constrain the SN explosion models as proposed in (Seitenzahl et al. 2009) has been used in other SNe Ia, for example SN 2012cg.

To recapitulate, based on the two-dimensional models, we aim at: (1) searching for WD parameters which corresponds to the general features of SN 2014J, (2) searching for appropriate He-detonation triggers which produce global asymmetries and qualitative features of SN 2014J, including large scale asymmetry and/or near-surface production of ^{56}Ni .

4.5. Constraints on Progenitor WD Mass

The ^{56}Ni mass observed in SN 2014J provides the important constraint on the progenitor WD mass. In Diehl et al. (2015)⁴, the ^{56}Ni mass is estimated to be $0.49 \pm 0.10 M_{\odot}$. From our simulation results in Table 1, we identify the possible mass range to be $M = 1.0 - 1.1 M_{\odot}$.

In Figure 6, we show the final ^{56}Ni mass against M for the four different detonation morphologies described in the previous section. To account for the observed ^{56}Ni mass in SN 2014J shown by the magenta box in Figure 6, we require $M = 0.95 - 1.00 M_{\odot}$ for the “S”-series and $M = 1.00 - 1.10 M_{\odot}$ for the “B”-series ($M_{\text{He}} = 0.10 M_{\odot}$). “D”- and “R”-series require $M = 1.05 - 1.10(1.00 - 1.05) M_{\odot}$ for $M_{\text{He}} = 0.05 (0.10) M_{\odot}$.

4.6. Constraints on Explosion Mechanisms

Another aspect to constrain SN 2014J is by the explosion geometry. The gamma-ray signal of SN 2014J has suggested the (near-)surface ^{56}Ni . We examine which initial detonation geometry allows the formation of ^{56}Ni at these regions. To extract the final distribution of ^{56}Ni , we use the tracer particle data when the ejecta

³ notice that in general a static WD is considered and the “poles” has a graphic meaning of being the upper and lower ends of the sphere, while the “equator” means the symmetry plane between the two “poles”.

⁴ Other works in the literature give a similar range. For example in Churazov et al. (2014) the upper limit can reach $0.7 M_{\odot}$, but the uncertainties are similar.

reaches homologous expansion and the structure of the ejecta is frozen out.

Future observations of SN 2014J, which will be able to disentangle the morphology, may thus provide clues to the initial configuration from measurements of different elements. For example, in Grefenstette et al. (2014, 2017) such a technique is pioneered in showing ^{44}Ti in Cas A to demonstrate how to disentangle and map the gamma-rays.

In Figure 3, we show the distributions of some representative elements for models with different detonation geometry. For example, for models starting with a He-detonation bubble, the Ni and Si ejecta are in a cocoon shape, compared to the spherical shape in the spherical detonation. The "B"-Type is also highly distinctive by the thick layer of the O-rich ejecta, compared to the spherical counterpart. On the other hand, "D"-Type and "R"-Type are different from the other two by the quasi-spherical O- and Si-rich ejecta, while the Ni-ejecta maintain an observable ellipticity. However, between D and R type models, Si and Ni show a similar distribution.

4.7. Constraints on the He Envelope Mass

We now examine the dependence of the large scale asymmetry on the He envelope mass. In Paper II we have presented a parameter survey on presented a parameter survey on the nucleosynthesis yield of SNe Ia using the sub-Chandrasekhar mass WD as the initial progenitor. It is shown that the He-envelope mass can strongly enhance the production of some Fe-peak isotopes, including ^{48}Ti , $^{50,51}\text{V}$ and ^{52}Cr .

In Figure 8, we plot the ^{56}Ni distribution for Models 100-050-R and 100-100-R. They differ by the mass of the He-envelope from 0.05 to 0.1 M_{\odot} . The distribution of ^{56}Ni after the explosion can show the existence of a large-scale asymmetry. In Model 100-100-R, with a more massive He envelope, the He-detonation is strong enough to drive the inwards propagation of the detonation to the core; this results in the complete disruption, where most of the matter in the CO core is spherical.

Therefore, this scenario will less likely exhibit a flickering as seen in SN 2014J. However, its ^{56}Ni distributes also close to the surface, which is an important feature of SN 2014J. As a result, the ejected ^{56}Ni around 10 degrees from the rotation axis has a higher radial velocity. The lower He-envelope mass makes the production of ^{56}Ni lower, and the synthesized ^{56}Ni is covered by the original He-envelope, which can block the gamma-rays emitted by the radioactive decay. This means a lower M_{He} model has more difficulties to reproduce the early gamma-rays observed from SN2014J.

Following the expansion of the ejecta, when the matter becomes optically thin to the gamma-ray, the inner structure of the ^{56}Ni and ^{56}Co distribution will be exposed. The velocity fluctuation, as seen from Figure 8, depending on the ejecta angle and their exact time to become optically thin, may coincide with the flickering feature as seen in SN 2014J.

4.8. Constraints from Nucleosynthesis

In Figure 9 we plot the ^{57}Ni and ^{56}Ni yields (prior to decay) of our simulated SN Ia models. Shown are models with $M_{\text{He}} = 0.05$ (solid lines) and $M_{\text{He}} = 0.10 M_{\odot}$ (dashed lines) at solar metallicity. We also show the SN 2014J data by the cross symbol (Yang et al. 2018). Apparently the models listed there are not sufficient to explain the high ^{57}Ni mass of this supernova.

Similarly, to demonstrate the effects of metallicity, we plot the dash-dotted line of Model 110-050-B for $Z = 0, 1, 3$ and $5 Z_{\odot}$ (corresponding to the data point from the bottom to the top). To explain the SN 2014J data, a model with $Z \sim 5 Z_{\odot}$ is required. All the trend lines for other models behave similarly. In Paper I we have shown that the metallicity has minor impact to the global explosion energetic. Instead, it shows its influence on the relative abundance ratio, in particular the high- Y_e isotopes including ^{54}Fe , ^{58}Ni and ^{55}Mn .

From this figure we observe that in order to explain the abundance pattern of SN 2014J, a high metallicity model with $Z \gtrsim 4 Z_{\odot}$ is necessary to be consistent with the high ^{57}Ni mass relative to ^{56}Ni .

5. EXTENSION TO OTHER SUPERNOVA OBSERVATIONS

5.1. Remnant Morphology

The morphology of the supernova ejecta and the shape of the SN remnant, such as Tycho (Ferrand et al. 2019), may directly link to the initial explosion configuration. By tracing the line emission of shock heated ejecta, the abundance of the measured elements can be revealed. In Seitenzahl et al. (2019) the tomography of three youngest SN Ia remnants, 0519-69.0, 0509-67.5 and N103B in the large Magellanic Cloud (Hughes et al. 1995) are studied for the first time. By examining the S-XII, Fe-IX and Fe-XV lines, they reconstructed the large-scale distributions of these elements in these remnants. These objects are sufficiently young such that the shocked-heated matter remains clearly visible and the shock front has not completely swept all the matter.

SN remnant 0519-69.0 has a more spherical shape, but with small scale perturbations on the surface as depicted by the X-rays. From our simulations, such feature is possible when we consider the Rayleigh-Taylor instabilities

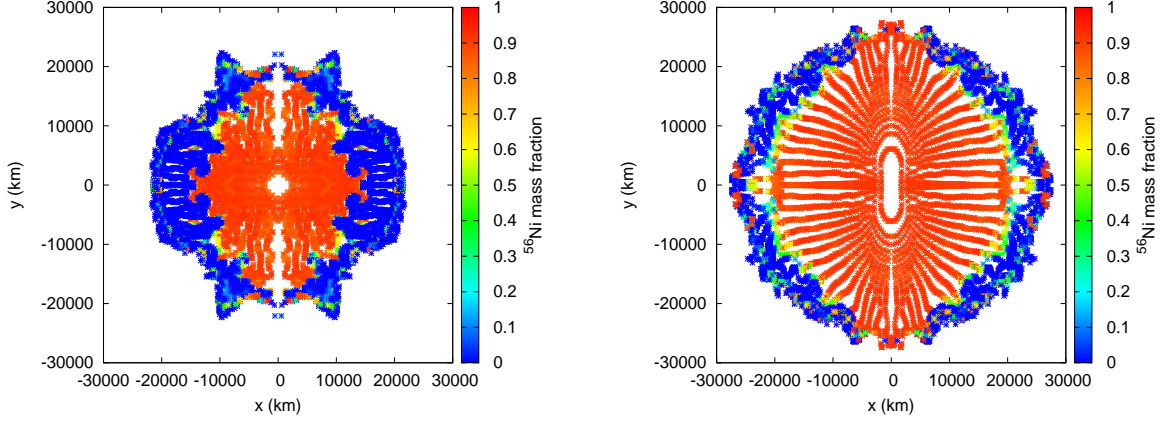


Figure 8. The ^{56}Ni mass fraction distribution of the explosion models 100-050-R (left panel) and 100-100-R (right panel).

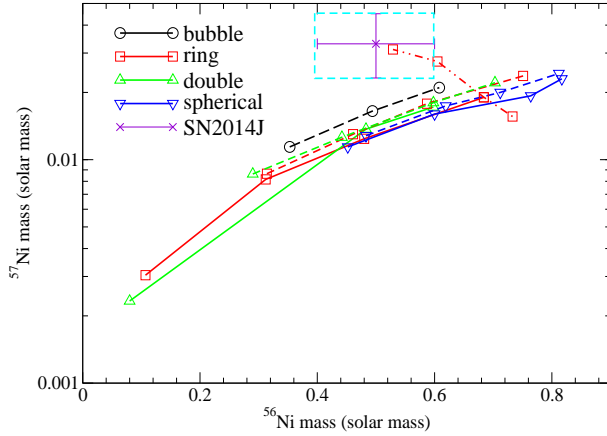


Figure 9. The ^{57}Ni mass against ^{56}Ni for the four different initial He-detonation configurations for $M_{\text{He}} = 0.05 M_{\odot}$ (solid line) and $0.10 M_{\odot}$ (dashed line) and for solar metallicity. The data point shows the observed $^{57}\text{Ni}/^{56}\text{Ni}$ mass ratio derived from the late-time light curve of the optical band. The red dash-dot-dot line corresponds to the sequence of Model 110-050-R50 using different initial metallicity of $Z = 0, 1, 3$ and $5 Z_{\odot}$ (from bottom to top). The data points from left to right corresponds to the models with $M = 0.9, 1.0, 1.05$ and $1.1 M_{\odot}$.

which freeze out during the expansion of the ejecta. As discussed in [Seitenzahl et al. \(2019\)](#), this remnant can be fitted by the Chandrasekhar mass WD of mass $1.4 M_{\odot}$. Their total Fe-mass is about $0.4 M_{\odot}$. This belongs to the lower side of SN Ia production ([Leung & Nomoto 2018](#)). Such features can be obtained for a higher mass progenitor $\sim 1.4 M_{\odot}$ with an initial central density $> 3 \times 10^9 \text{ g cm}^{-3}$.

SN remnant 0509-67.5 also has a spherical shape where the ejecta including X-ray demonstrates a close-to-spherical structure. The estimated mass of this remnant is $\sim 1.0 M_{\odot}$ with $0.15 M_{\odot}$ He. To achieve such spherical shape, a spherical He-detonation is necessary to avoid any large-scale asymmetry created during the

C-detonation. In fact, when such a heavy He-envelope is included, spherical detonation is more preferred because the expected nuclear runaway time can easily be shorter than the convection time scale, thus triggering simultaneous burning in the spherical layer near the He-CO interface. Also, as discussed in Paper II, the thick He envelope $> 0.1 M_{\odot}$ can bring a very severe excess in light iron-peak elements such as Ti, V and Cr. Future detection of these elements may provide further confirmation in this explosion picture. At last, its total Fe mass $\sim 0.5 M_{\odot}$ can be mapped consistently to our models such as 100-100-S.

SN remnant N103B also has an aspherical ejecta shape. However, not much analysis of this object is reported in that work. Despite that, from the morphology of ejecta, an arrow or cone shape of Fe XIV distribution can be observed. Such an aspherical shape with such a pointing effect can indicate the focused shock in the CO core and its consecutive breakout, as hinted from Figure 2. In our work, due to the reflection symmetry, the cone shape is always paired on both sides. We expect that, if we allow single bubble without assuming reflection symmetry, a one-sided feature can be resulted as done in more general simulation such as [Tanikawa et al. \(2019\)](#); [Gronow et al. \(2020\)](#).

5.2. Remnant Element Abundance

Another possibility to trace the remnant progenitor is to examine the abundance patterns. As described in previous sections, the X-ray spectra from SNR can provide essential clues on the relative amount of elements, especially iron-peak elements including Cr, Mn, Fe and Ni ([Yamaguchi et al. 2015](#); [Martínez-Rodríguez et al. 2017](#)). They can directly constrain from which type of WD progenitor and the explosion physics. In their works, the SNR spectra including Tycho, Kepler, 3C 397, and G337.2-0.7 in the Milky Way and N103B in

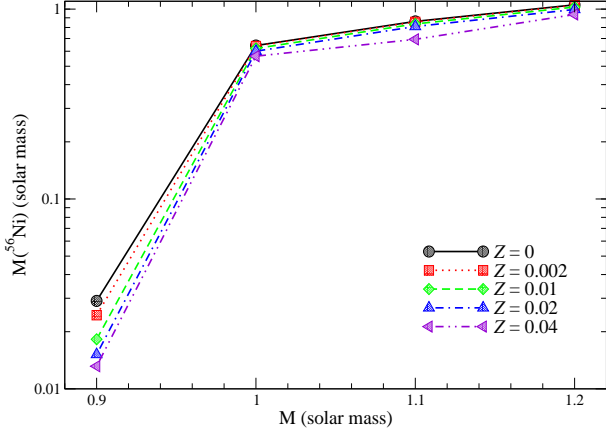


Figure 10. ^{56}Ni production for the sub-Chandrasekhar mass models with initial spherical detonation structure for $M = 0.9$ to $1.2 M_{\odot}$ at $Z = 0, 0.002, 0.01, 0.02$ and 0.04 .

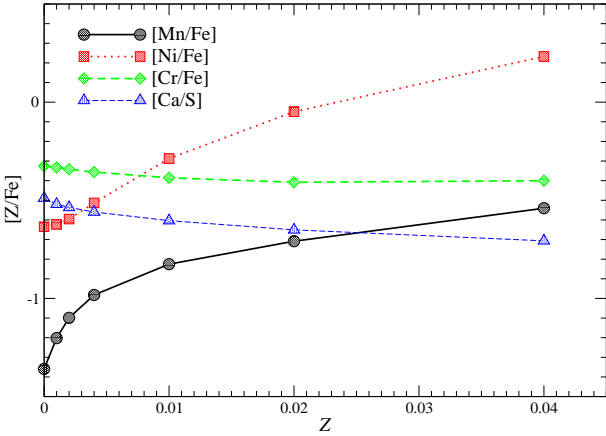


Figure 11. Abundance ratios for the sub-Chandrasekhar mass model with initial spherical detonation structure for $M = 1.0 M_{\odot}$ at $Z = 0, 0.002, 0.01, 0.02$ and 0.04 .

the Large Magellanic Cloud are analyzed. It is suggested that to explain the variety of remnant chemical abundance pattern, factor beyond neutronization (i.e., metallicity tracer by ^{22}Ne) is necessary. Here we examine how the metallicity affects the elemental distribution and ^{56}Ni .

In Figure 10 we plot the mass and Z dependence of ^{56}Ni production for the sub-Chandrasekhar mass WD models with $M = 0.9 - 1.2 M_{\odot}$ and $Z = 0 - 0.04$. The ^{56}Ni -production is strongly M -dependent and monotonically increasing with M . The transition from complete burning to incomplete burning shows the strongest effects between $0.9 - 1.0 M_{\odot}$ by a change of almost a factor of ten within this mass range. The effects of metallicity can be seen but is small compared to the effects of M . In general a $\sim 10\%$ difference can be seen among the Z -range examined here.

In Figure 11, we plot the mass fraction ratio of $[\text{Mn}/\text{Fe}]$, $[\text{Ni}/\text{Fe}]$, $[\text{Cr}/\text{Fe}]$ and $[\text{Ca}/\text{S}]$ for our sub-Chandrasekhar mass WD models with a mass $1.0 M_{\odot}$ for metallicity $Z = 0, 0.002, 0.01, 0.02$ and 0.04 . The metallicity effect is much larger. Major IPEs such as $[\text{Mn}/\text{Fe}]$ and $[\text{Ni}/\text{Fe}]$ differ by ~ 0.6 dex between models with $Z = 0$ and $Z = 0.04$. Both of them increase when Z increases. On the other hand, minor IPEs such as $[\text{Cr}/\text{Fe}]$ and IMEs $[\text{Ca}/\text{S}]$ show much smaller variations by only 0.1 dex in the Z -range examined. They decrease when Z increases.

However, we remark that the exact values of $[\text{Ca}/\text{S}]$ and $[\text{Cr}/\text{Fe}]$ are more prone to systematic uncertainties. It is because the light IPEs including Ca and Cr can be produced in both NSE and incomplete-Si burning. For Mn, Fe and Ni, they are robustly produced in NSE matter, where complete and instantaneous energy release can be assumed. On the other hand, for Ca and Cr, the numerical scheme in how the low density burning can affect the formation of these elements. For the same amount of energy release, a longer energy deposition time can slow down the thermal expansion. As a result, the matter has more time to carry out slow nuclear reaction in the α -rich freezeout regime before the ejecta expands and becomes too cold for any significant nuclear reactions. The case for low density matter contains complication because the actual reaction depends on the detailed chemical composition, which is not well traced in multi-dimensional composition. The simplified chemical composition (7-isotope network) may not provide an accurate estimation in how fast those reactions and the associated energy production take place.

6. DISCUSSION

6.1. Connection to the Works in the Literature

Our work has suggested that the He-detonation is the key feature for explaining some well observed SNe Ia, such as SN 2014J. However, it also remains to be understood in details how the ignition of surface He can be coupled to the current explosion mechanism which ignites carbon centrally in the WD. Three-dimensional Low-Mach number hydrodynamical simulations of the He-burning envelope is necessary for realizing how the initial hydrostatic He-burning develops into nuclear runaway. The detonation size is typically assumed to be as large as the pressure scale height (Bildsten et al. 2007; Shen & Bildsten 2009; Dan et al. 2014). On the other hand, temperature fluctuations tend to favour the ignition at a single spot, which has a much smaller size. A stringent limit on the temperature fluctuation appears because of its small size (Holcomb et al. 2013). Contamination of C-rich matter from the CO-core can be

an alternative to decrease the necessary size of nuclear burning (Shen & Moore 2014).

The pre-runaway phase of the double detonation model is unclear until recent large scale works on clarifying the possibility of such proposal (Jacobs et al. 2016). Depending on the convection flow, different He-runaway pattern scenarios could occur from the most non-spherical extreme, i.e. a bubble, to the most spherically symmetric case. The modeling of such process is typically much longer than the hydrodynamics timescale, in order to capture the first nuclear runaway from nuclear reactions directly. To resolve the first runaway, the simulation requires the hydrodynamics timescale t_{hyd} to be smaller than the timescale of nuclear burning t_{burn} and convection t_{conv} , i.e. $t_{\text{hyd}} < t_{\text{burn}} < t_{\text{conv}}$ (Glasner et al. 2018). Only recently there are a few pioneering models using hydrodynamics simulations in the low-Mach number regime to follow how the convection develops into nuclear runaway self-consistently (Zingale et al. 2013; Jacobs et al. 2016). Therefore, to trace the possible origin and to explain the observations of SN 2014J, we considered different possibilities in how the He-detonation can be triggered.

On the other hand, once a detonation spot forms, the second detonation is in general inevitable. Recent three-dimensional large-scale hydrodynamics simulations of one- or multi-spot He detonation has been found to be robust in triggering the off-center detonation by geometric convergence in a quiet He envelope (Moll & Woosley 2013). The required He envelope mass required to trigger the second detonation can drop significantly from $\sim 0.1 M_{\odot}$ to $\sim 10^{-3} M_{\odot}$ for a WD mass increasing from 0.8 to 1.3 M_{\odot} (Fink et al. 2010), while observationally a He envelope below 0.05 M_{\odot} is favoured due to the discrepancy with the theoretical light curve when a massive He envelope is applied (Woosley & Kasen 2011).

Such diversities in the He detonation trigger and detonation wave interactions have provided the flexibility to account for the diversity of SNe Ia. In Diehl et al. (2015), the multiple-plume structure is proposed to illustrate the apparent flickering of the ^{56}Co decay line. Based on our models, it is possible that such feature can be realized by multiple spots in the He envelope. For a quantitative comparison, three-dimensional models are required for a one-one matching of the observables and predicted signatures. Nevertheless, using the virtue that geometric convergence and laminar wave shock collision do not differentiate between two- and three-dimensional simulations, our models can shed light on what kind of shock interaction, and hence what kind of detonation pattern, are necessary for reproducing features taken from SN 2014J.

6.2. How Typical is SN 2014J?

In this work we explored the possible triggering and ignition mechanisms that might lead to the asymmetric properties demonstrated by the observational features of SN 2014J. Among all models, the closest model we obtain is Model 110-050-R with a metallicity $\gtrsim 4 Z_{\odot}$.

The total (WD) mass required is 1.0 – 1.1 M_{\odot} , at the intermediate to high mass end of sub-Chandrasekhar mass WD. Stellar evolution theory suggests a progenitor mass constrained in a range likely to be $\sim 6 - 7 M_{\odot}$ (See e.g. Catalán et al. (2008); Doherty et al. (2015) for the progenitor-final mass relation)⁵. For a sub-Chandrasekhar mass WD to produce 0.6 M_{\odot} ^{56}Ni as a normal SN Ia, theory suggests an initial mass of $\sim 1.0 M_{\odot}$ in one-dimensional models (Shigeyama et al. 1992; Nomoto & Leung 2018; Shen et al. 2018). The ^{56}Ni mass production is known to be sensitive to the progenitor mass because of the density dependence of ^{56}Ni production, with a minimum $> 5 \times 10^7 \text{ g cm}^{-3}$. Hence, a higher WD mass not only ensures a higher central density, but also a higher energy release at the center, which favours the propagation of the detonation wave. In the one-dimensional models, the 1.1 M_{\odot} case gives rise to a bright SN Ia for its $\sim 0.8 M_{\odot}$ production of ^{56}Ni . However, in multi-dimensional models there are variations in the ^{56}Ni production based on the initial He-detonation structure. The aspherical He-detonation tends to give a lower ^{56}Ni due to off-center ignition of C+O detonation because of the density gradient as discussed in previous sections.

The He envelope mass required by our calculations is $\sim 0.05 M_{\odot}$. This is a marginal value for the He detonation to be observed (Woosley & Kasen 2011), where the optical observational features remains compatible with the normal SN Ia data. A high He envelope mass likely overproduces some iron-peak elements including Cr and V near the surface. This changes the typical isotope and element abundance distributions compared to normal SNe Ia where such elements are produced in a deeper layer, e.g., the Chandrasekhar mass WD with deflagration-detonation transition. Such high opacity material in the envelope may make the explosion appear redder in optical spectra (Polin et al. 2019). However, their results are based on a one-dimensional model where the He-rich matter is always burnt from high den-

⁵ However, we remark that at about 7 M_{\odot} the final remnant mass is close to the transition mass of CO WD, where it is possible the core may have undergone advanced burning which destroys ^{12}C and produced ^{20}Ne , leaving a hybrid O+Ne+Mg core with a C+O envelope. The exact transition mass depends on the stellar evolution code and input physics.

sity to low density. Therefore, the high density matter has always a longer time to carry out nuclear reactions, which favors the production of such iron-peak elements. However, for aspherical detonation, this is not always true. The Cr and V production depends on how the He-detonation spread around the He-envelope.

Another theoretical uncertainty is the exact He mass when the first nuclear runaway starts. The exact He envelope mass depends on the mass accretion rate and the type of binary system (single or double degenerate). A higher M_{He} is more likely from the double degenerate scenario while a lower M_{He} is more likely from the single degenerate scenario. Such calculation has been done by Kawai et al. (1988) from the stellar evolutionary perspective. The steady state accretion of He on C+O and O+Ne+Mg WDs are investigated in the single degenerate scenario. It is shown that the He envelope mass drops sharply with the C+O core mass, with $\sim 10^{-2} M_{\odot}$ for a $0.7 M_{\odot}$ C+O core down to $\sim 10^{-6} M_{\odot}$ for a $1.36 M_{\odot}$ core. This shows that the steady state accretion in the single degenerate scenario may not provide a robust way for accumulating a He envelope beyond $10^{-2} M_{\odot}$ in a WD of mass $1.00 M_{\odot}$ or above.

The detonation required at the beginning is from a ring around the "equator". Such a configuration is shown to produce more aspherical features in the Ni distribution, which would be compatible with the multiple redshifted ^{56}Co decay lines measured in SN 2014J. But, we have also found that other types of initial detonation such as the "D"-Type and the "B"-Type, may also produce similar characteristics, although less pronounced. How the He detonation is initialized is a matter of debate.

For a one-dimensional model, an entire mass shell is ignited simultaneously because of the assumed symmetry. However, it is unclear whether such symmetry can be maintained prior to the ignition. For example, in the single degenerate scenario, the accretion of matter from the companion star through Roche Lobe overflow has in general a high angular momentum. Such rapidly rotating matter, when accumulated on the stellar surface, may create strong dragging, which disturbs the material near surface. Also, He burning near the core-envelope interface may trigger convective motion (see e.g. Jacobs et al. 2016). This creates a highly turbulent background due to the shear between the quasi-static C+O core and the rapidly rotating He-rich matter.

Knowing that the runaway of He is highly temperature-sensitive, it is conceivable that the ignition may occur at random locations and spherical symmetry is broken. In the most extreme case, only one spot can be ignited, which corresponds to the "B"-Type ex-

plosion. If the rotation symmetry may be preserved, then the "R"-Type explosion is one of the possibilities. However, the exact configuration will be the best estimated from the detailed multi-dimensional hydrodynamics simulations for the last minutes before the runaway, to capture how all these processes interfere with each other (refer e.g. Zingale et al. 2011; Malone et al. 2014).

The early ^{56}Ni signal can act as a tracer to the explosion mechanism. In Figure 5 we show that how the He-detonation is initiated can strongly influence the surface ejecta composition and its angular dependence. In particular, the early low redshift ^{56}Ni line implies the possibility that we are observing SN 2014J close to where shock is initialized. If we observe the shock converging point, the abundant elements of ^{28}Si , ^4He may easily block the gamma-ray. The later ^{56}Co line can have an origin from multiple shock convergence on the He-envelope. This can be triggered by for example multiple rings or bubbles with non-uniform orientation. However, the exact details may require future study because the multiple plume feature (Diehl et al. 2015) indicates multiple shock convergence history and different locations for triggering the

Our explorations favour a high metallicity of the WD compared to the solar metallicity. In fact, this feature is common to the observed SNe Ia whose chemical abundance is extracted from their light curve and spectra. Such high metallicity appears to be common in recently observed SNe Ia, e.g. SN 2012cg (Graur et al. 2016; Leung & Nomoto 2018, 2020a), and SN Ia remnant 3C 397 (Yamaguchi et al. 2015; Leung & Nomoto 2018). These works have demonstrated that a super-solar metallicity is paramount to boost certain isotope or element ratios, and especially the $^{57}\text{Ni}/^{56}\text{Ni}$ or Mn/Ni ratios. Such effects cannot be completely replaced by tuning other major parameter such as the white dwarf mass, M , or the nuclear runaway structure. These examples demonstrated that metallicity of exploding WDs can be higher than solar metallicity. A detailed evolution path of such high metallicity WD progenitor would be an interesting future work.

6.3. Dependence on Model Dimensionality

We note that there exist controversies regarding two-dimensional modeling containing symmetries, which might not necessarily be realized in reality. Ideally, three-dimensional models are required to provide a comprehensive and self-consistent explanation to match the explanation in a one-one correspondence. Here we briefly recapitulate how we use two-dimensional models and why this can still provide reliable estimates.

First, two-dimensional models allow more time-effective search of appropriate models. As indicated in our previous works (Papers I and II), the parameter space which is suitable for SNe Ia is large. The running time for one n -dimensional hydrodynamical model scales with N^n for a grid mesh of N^n . Typical resolution requires ~ 500 grids for one direction. This means three dimensional models are at minimum ~ 500 times more computationally expensive. It takes in general three to five days for our two-dimensional model to complete its hydrodynamical simulation and its nucleosynthesis. This simple scaling implies that a three-dimensional model requires at minimum months for a single model. This is beyond the computational time we can afford for a practical model investigation.

Second, the large-scale aspherical effect can be well captured by two-dimensional models. We remind that 2D-models are capable of producing three-dimensional aspherical structure such as a bubble or a ring naturally, and also the one-dimensional spherical structure. Three-dimensional simulations can produce more complex structures in the form of multiple bubbles, for example. In fact, the processes determining how the second detonation starts depends on the wave collision details. These can already be captured by one of the two-dimensional scenarios. Furthermore, two-dimensional models provide the minimum perturbation from the spherical symmetry. Notice that SN 2014J has features which can be explained by the classical W7 model on a broader picture, even though having aspherical features. meanwhile. Therefore, it might be considered as a generalization to start from models which behave almost spherical, and extend them to lower symmetries.

Third, the symmetry is conserved in the simulation. As demonstrated in Moll & Woosley (2013), where one-, two- and three-dimensional simulations of the sub-Chandrasekhar mass models are carried out, one of their explicit three dimensional models with a two-dimensional counterpart gives agreeing results with each other. This provides some support that in the explosion phase, symmetry does not break during its propagation. Similar three-dimensional models in Gronow et al. (2020) also demonstrated similar features, in that the detonation propagates like a two-dimensional front. Furthermore, in this work, we further show that the two-dimensional spherical models explode spherically as in the one-dimensional case. Both results support that our axis-symmetric model remains to be axis-symmetric throughout the simulations, as long as turbulent motion is unimportant. This is true during the explosion phase (~ 1 s). When time is sufficiently long, i.e. in the nebular phase, we expect that the Rayleigh-Taylor instabil-

ities play a role and perturb the morphology. Then, initial seeds break the rotation symmetry. However, such effects are secondary compared to the large-scale asymmetry and require more time to grow. Also, similar to models in the literature (see e.g. Fink et al. 2007; Sim et al. 2010; Fink et al. 2010), we set up WDs in hydrostatic equilibrium as the initial condition. The quiet environment suggests that the turbulent motion is suppressed.

Finally, in this work we focus on the common features which exist in both two- and three-dimensional models. In particular we investigate how shock waves superpose, interact with each other, or grow by themselves through geometric convergence, and consequently generate the structure that breaks the spherical symmetry. As indicated in Paper II, how the wave interacts are independent of the boundary condition. Thus, our two-dimensional model can offer the necessary starting point to explore which kind of detonation structure is necessary to generate the corresponding interaction for creating the large-scale asymmetry.

6.4. Conclusion

In this article we explored the parameter space in the classical double detonation model which can produce observables indicating deviations from spherical symmetry. We studied how the initial detonation geometry affects the final explosion morphology by examining the ejecta composition in the spatial distribution and velocity space. We studied how the spherical symmetry can be broken for creating large-scale asymmetry. The sub-Chandrasekhar mass WD progenitor tends to produce more pronounced asymmetry than the Chandrasekhar mass WD progenitor. The surface He detonation can be the origin of the early ^{56}Ni gamma-ray line of some SNe Ia, e.g. SN 2014J and the recently observed early bumps in the observed light curves of some SNe Ia.

We have examined how the initial mass, He-detonation geometry, affects the final explosion results, in particular the ejecta geometry and element distribution in both spatial and velocity phase space. We observe that starting the He-detonation as a bubble (with lowest symmetry), to a ring, and then a sphere (with highest symmetry), may give observable differences in the ejecta morphology, ejecta velocity for the characteristics elements, including He, O, Si, S, Fe, and Ni, and their directional dependence.

We have provided a detailed case study on searching for models which may resemble with the qualitative features observed in SN 2014J based on the gamma-ray line detections and the late-time photometry of the optical band. Four key aspects of SNe Ia explosion are:

- (1) The total mass M of the WD determines the total ^{56}Ni production;
- (2) The He-envelope mass M_{He} determines the large scale asymmetry in the radial distribution of ^{56}Ni ;
- (3) The metallicity determines the required $^{57}\text{Ni}/^{56}\text{Ni}$ mass;
- (4) The initial He-runaway geometry determines the surface ^{56}Ni distribution.

From our explorations simulating a set of key scenarios, we conclude that the SN 2014J progenitor should have the following properties:

- (1) An initial He-detonation in the orbital plane set by the binary companion;
- (2) a WD mass in the range from 1.00 to 1.10 M_{\odot} ;
- (3) a WD metallicity in the range from 3 to 5 Z_{\odot} ;
- (4) a He envelope mass $\sim 0.05 - 0.10 M_{\odot}$.

We also derived the detailed velocity distributions of some major isotopes, for example ^{16}O , ^{28}Si , ^{54}Fe and $^{56-58}\text{Ni}$ and the spatial distributions of major IPEs including Cr, Mn, Fe and Ni. Future observations of the ejecta morphology by specific elements (e.g. Seitenzahl et al. (2019)) can provide a strong constraint on the models presented in this work. Large-scale features in these objects might reveal how the detonation

has interacted during its propagation, thus shedding light on its initial detonation pattern.

At last we discuss the recent application of SN tomography as presented in Seitenzahl et al. (2019) for the SN remnants 0519-69.0, 0509-67.5 and N103B. From how aspherical the SN ejecta in the reverse-shock heated region, and their corresponding Fe-mass are, we can deduce the fundamental properties of the progenitor including whether it is a Chandrasekhar or sub-Chandrasekhar WD, the expected initial mass and detonation geometry. We also summarize the M - and Z -dependence of the major element ratios typically found in spectra of SN remnants.

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APPENDIX

A. WHITE DWARF MODELS FOR TYPE IA SUPERNOVA

Type Ia supernovae (SNe Ia) are the thermonuclear explosions of CO white dwarfs (WDs) (see e.g. Hillebrandt & Niemeyer 2000; Nomoto & Leung 2018). A single CO WD does not spontaneously undergo nuclear burning. In a close binary system, on the other hand, the WD gains mass by mass transfer from its companion star, which includes a slightly evolved near main-sequence star, a red-giant, and a He-star (single degenerate scenario, e.g., Nomoto 1982b; Kawai et al. 1988) or a WD (double degenerate scenario Iben & Tutukov 1984; Webbink 1984).

During the mass accretion, if the accretion rate is relatively low, the accreted He is accumulated on the surface and eventually He-burning is ignited first in the off-center hot spot when the WD mass is the sub-Chandrasekhar mass (Nomoto 1982b). Then He-detonation is developed (Nomoto 1982a).

If the accretion rate is relatively high, on the other hand, the WD mass reaches the near-Chandrasekhar mass and the central temperature exceeds $\sim 3 \times 10^8$ K. Then the energy generation rate of ^{12}C -burning exceeds the neutrino cooling rate. ^{12}C -burning is unstable to develop a C-deflagration in the center (Nomoto 1982b) ⁶.

A.1. Near-Chandrasekhar Mass White Dwarf Models

When ^{12}C -burning is ignited in the center of the near-Chandrasekhar mass WD, the central density is as high as $2 - 3 \times 10^9$ g cm⁻³. Electrons are strongly degenerate, so that the gas pressure is not so sensitive to the temperature. The temperature rise becomes unregulated as the carbon burning rate is strongly temperature sensitive ($\sim T^{33}$). This sets the first seed of nuclear runaway. Simultaneously, the rapid temperature rise does not trigger a shock

⁶ Depending on the mass accretion rate, the white dwarf can undergo centered deflagration, double detonation or accretion-induced collapse (see also Nomoto 1982a; Nomoto et al. 1984, for a detailed phase diagram)

because the pressure growth is small compared to the temperature rise due to the strong degeneracy even when the central temperature becomes as high as $\sim 10^{10}$ K (Nomoto et al. 1976; Nomoto & Sugimoto 1977). As a result, the temperature jump becomes a localized event, where the temperature discontinuity propagates by microscopic processes instead of macroscopic shock compression. Thus, the initial runaway is likely to be in the form of a deflagration wave (Nomoto et al. 1976; Nomoto et al. 1984). The short mean free path of electron conduction in such a density implies very thin flame front ($\sim 10^{-3}$ cm) compared to the size of a WD ($\sim 10^3$ km) (Timmes & Woosley 1992).

Despite the turbulent motion emerges down to the Kolmogorov scale (10^{-3} cm assuming a typical Reynolds number of $\sim 10^{14}$), the Gibson scale decreases with density and it is in general larger than the Kolmogorov scale. Flame structure with a size below the Gibson scale ($\sim 10^{-1}$ km at the center), is smoothed (Niemeyer & Hillebrandt 1995; Röpke et al. 2003, 2004a), albeit eddies can appear below this scale.

If the propagation speed of the subsonic deflagration is fast enough, the deflagration efficiently releases the necessary energy for unbinding a WD and creates a successful explosion like W7 model (Nomoto et al. 1984) as also does the detonation wave (Arnett 1969).

The explosive nuclear burning at high densities synthesize iron-peak elements (e.g., Thielemann et al. 1986; Iwamoto et al. 1999). However, the observed intermediate mass elements should be synthesized at lower densities, which suggests the explosion consists of a subsonic burning, i.e. deflagration (Nomoto et al. 1976; Nomoto & Sugimoto 1977), which decreases the densities at the flame front.

If the propagation of the deflagration wave is slow, it may not unbind the star (but in some cases may cause pulsation) because the stellar expansion makes the deflagration wave quench (Nomoto et al. 1976). Subsequent transition from deflagration to detonation is vital for explaining a successful SN Ia explosion, known as the deflagration-detonation transition (Nomoto et al. 1984; Khokhlov 1991; Arnett & Livne 1994a,b; Iwamoto et al. 1999; Gamezo et al. 2003, 2004; Röpke et al. 2007). However, the deflagration-detonation transition requires a turbulence strength which is less likely to be reached (see, e.g., Khokhlov et al. 1997; Niemeyer et al. 1999; Lisewski et al. 2000; Gamezo et al. 2005; Woosley et al. 2009). In order to realize this effect, ab initio numerical experiments with very fine resolutions (~ 0.1 km) are necessary (Kushnir et al. 2012), which are one to two orders of magnitude below the affordable resolution. Recent direct experiments, both numerical and laboratory ones using methane-air mixture resolving turbulent motion explicitly, have demonstrated that turbulent acceleration can be an important key factor (Poludnenko et al. 2011; Poludnenko et al. 2019).

The motion of the deflagration wave can be convoluted. The subsonic propagation ($\sim 1\%$ speed of sound) implies that the deflagration wave structure is coupled with the underlying fluid motion, which means that the flame structure is also susceptible to various hydrodynamics instabilities such as the Rayleigh-Taylor instabilities (Bell et al. 2004a; Zingale et al. 2005; Hicks 2015, 2019), Kelvin-Helmholtz instabilities, Landau-Derrius instabilities (Bell et al. 2004b; Röpke et al. 2004b) and pulsational instabilities (Glazyrin 2013; Glazyrin et al. 2013; Glazyrin 2014; Poludnenko 2015). On the contrary, the supersonic detonation is less sensitive to fluid motion. However, direct numerical simulations of the small scale detonation shows spontaneous cellular structure formation behind the detonation wave front (Gamezo et al. 1999).

To model the explosion, following how the deflagration propagates reveals how the energy is released. The sub-grid scale of the reaction front indicates that on-site modeling is inaccurate, but a sub-grid scale model is necessary to describe partial cell burning and irregular wave front inside the cell. This relies on the sub-grid scale turbulence model (Clement 1993; Shih et al. 1995a,b; Niemeyer & Hillebrandt 1995) and the flame tracking scheme.

The sub-grid scale turbulence model assumes that eddy motion below the resolved scale can be well described by statistical models. This gives an accurate approximation given the large difference between the resolved scale and the much smaller Kolmogorov's scale. Scaling relation has been studied explicitly in direct simulation (see e.g. Fisher et al. 2019). The model tracks the generation and dissipation of eddy motions by channels including shear-stress, compression, Rayleigh-Taylor instabilities and so on (see e.g. Shih et al. 1995a,b; Niemeyer & Hillebrandt 1995; Schmidt et al. 2006).

Flame tracking schemes are algorithms designed for resolving sub-grid scale features. There are multiple representations, including (1) the advective-diffusive-reactive equations (Khokhlov 1995; Vladimirova et al. 2006; Townsley et al. 2007), (2) level-set methods (Osher & Sethian 1988; Sethian 2001), and (3) point-set methods (Glimm et al. 1999, 2000; Zhang 2009; Leung et al. 2015a). The main idea is to introduce additional variables with model parameters which represent how much the grid is partially burnt, from that the actual flame front geometry is reconstructed.

A.2. Sub-Chandrasekhar Mass White Dwarf Models

In a sub-Chandrasekhar mass WD, the less degenerate matter with a lower density means that the detonation is more likely (Nomoto 1982b). Such scenario is viable when the surface energy production is faster than its heat loss by convection or expansion (Jacobs et al. 2016). The initial nuclear runaway can be triggered by accretion from its companion star in the single degenerate scenario, or through a violent merger in the double degenerate scenario (see, e.g. Tanikawa et al. 2015, 2019). Pure CO matter has a high ignition threshold for its high temperature ($\sim 1 - 2 \times 10^9$ K) (e.g., Sato et al. 2015), which is shown to be difficult to trigger and sensitive to the way of contact (Dan et al. 2012). WD merger with a helium envelope can suppress this ignition condition, but its required hotspot size can be non-realizable in a thin white dwarf envelope (see e.g. Shen & Bildsten 2009). Mixing with C/O-rich matter through for example turbulence can resolve this difficulty (Holcomb et al. 2013; Piro 2015). The violent merger of two CO white dwarfs (Pakmor et al. 2012) are therefore challenging for a robust ignition as the collision can fail to generate the spots, sufficiently hot for the first runaway (Dan et al. 2012, 2014). Mixture with helium provided by its He-envelope or the companion star lowers the ignition temperature such that the detonation trigger is less sensitive to the merging dynamics (Shen & Moore 2014). A thin layer of He ($\sim 0.01 M_\odot$) can already trigger the second explosion more robustly (Pakmor et al. 2013). When the He-detonation fails to trigger the second detonation. The star develops like a nova and explodes as a so-called Type "Ia" supernova (Bildsten et al. 2007; Shen et al. 2010; Waldman et al. 2011).

Even when the C-detonation trigger becomes robust with the aid of He, the exact position and timing of the detonation are unclear because they depend on the dynamics of the He-atmosphere. These require multi-dimensional low Mach number simulations of the atmosphere for multiple eddy turnover time. Multiple possibilities exist including direct He-ignition, C-ignition, or ignition after the merger process when a Chandrasekhar mass WD is formed (Dan et al. 2011; Shen et al. 2018; Tanikawa et al. 2019). Geometric convergence in a low-mass WD is more difficult to achieve (Shen & Moore 2014). High resolution simulations using ab initio approach is necessary to trace when and where the first hot spot appears (Fenn & Plewa 2017). The asymmetry in a three-dimensional simulation tends to suppress the prompt detonation as the geometrical convergence breaks down in the violent merger scenario (Fenn et al. 2016).

In a low mass WD, the detonation front has a size comparable with the resolved scale. This allows directly coupling of the hydrodynamics with a nuclear reaction network (Shen et al. 2018; Polin et al. 2019). In a more massive CO WD (central density $\geq 10^8$ g cm $^{-3}$), the detonation width can be much smaller than the resolved grid size (~ 10 km). sub-grid scale methods or adaptive mesh refinement are often used in the literature.

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